Ludwig
Wittgenstein

Tractatus Logico-Philosophicus

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With an introduction by Bertrand Russell



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10. Wittgenstein: Tractatus, 4.1–5.5 and 6.1s

we can indicate a point on the paper even if we do not know what black and white are, but if a proposition has no sense, nothing corresponds to it, since it does not designate a thing (a truth-value) which might have properties called 'false' or 'true'. The verb of a proposition is not 'is true' or 'is false', as Frege thought: rather, that which 'is true' must already contain the verb.

Against Frege: 'is true' isn't the verb. What's true already contains it.

4.064 Every proposition must *already* have a sense: it cannot be given a sense by affirmation. Indeed its sense is just what is affirmed. And the same applies to negation, etc.

Sense comes first: affirming can't give it, since the sense is what's affirmed.

4.0641 One could say that negation must be related to the logical place determined by the negated proposition.

The negating proposition determines a logical place *different* from that of the negated proposition.

The negating proposition determines a logical place with the help of the logical place of the negated proposition. For it describes it as lying outside the latter's logical place.

The negated proposition can be negated again, and this in itself shows that what is negated is already a proposition, and not merely something that is preliminary to a proposition.

4.1 Propositions represent the existence and non-existence of states of affairs.

4.11 The totality of true propositions is the whole of natural science (or the whole corpus of the natural sciences).

4.111 Philosophy is not one of the natural sciences.

(The word 'philosophy' must mean something whose place is above or below the natural sciences, not beside them.)

4.112 Philosophy aims at the logical clarification of thoughts.

Philosophy is not a body of doctrine but an activity.

A philosophical work consists essentially of elucidations.

Philosophy does not result in 'philosophical propositions', but rather in the clarification of propositions.

Without philosophy thoughts are, as it were, cloudy and indistinct: its task is to make them clear and to give them sharp boundaries.

4.1121 Psychology is no more closely related to philosophy than any other natural science.

Theory of knowledge is the philosophy of psychology.

Does not my study of sign-language correspond to the study of thought-processes, which philosophers used to consider so essential to the philosophy of logic? Only in most cases they got entangled in unessential psychological investigations, and with my method too there is an analogous risk.

4.1122 Darwin's theory has no more to do with philosophy than any other hypothesis in natural science.

4.113 Philosophy sets limits to the much disputed sphere of natural science.

4.114 It must set limits to what can be thought; and, in doing so, to what cannot be thought.

It must set limits to what cannot be thought by working outwards through what can be thought.

4.115 It will signify what cannot be said, by presenting clearly what can be said.

4.116 Everything that can be thought at all can be thought clearly. Everything that can be put into words can be put clearly.

4.12 Propositions can represent the whole of reality, but they

cannot represent what they must have in common with reality in order to be able to represent it—logical form.

To represent logical form we'd have to stand outside logic. Propositions mirror it instead.

In order to be able to represent logical form, we should have to be able to station ourselves with propositions somewhere outside logic, that is to say outside the world.

4.121 Propositions cannot represent logical form: it is mirrored in them.

What finds its reflection in language, language cannot represent.

What expresses itself in language, we cannot express by means of language.

Propositions show the logical form of reality.

They display it.

4.1211 Thus one proposition '*fa*' shows that the object *a* occurs in its sense, two propositions '*fa*' and '*ga*' show that the same object is mentioned in both of them.

If two propositions contradict one another, then their structure shows it; the same is true if one of them follows from the other. And so on.

4.1212 What can be shown, cannot be said.

What can be shown can't be said.

4.1213 Now, too, we understand our feeling that once we have a sign-language in which everything is all right, we already have a correct logical point of view.

4.122 In a certain sense we can talk about formal properties of objects and states of affairs, or, in the case of facts, about structural properties: and in the same sense about formal relations and structural relations.

(Instead of 'structural property' I also say 'internal property'; instead of 'structural relation', 'internal relation'.

I introduce these expressions in order to indicate the source of the confusion between internal relations and

relations proper (external relations), which is very widespread among philosophers.)

It is impossible, however, to assert by means of propositions that such internal properties and relations obtain: rather, this makes itself manifest in the propositions that represent the relevant states of affairs and are concerned with the relevant objects.

4.1221 An internal property of a fact can also be called a feature of that fact (in the sense in which we speak of facial features, for example).

4.123 A property is internal if it is unthinkable that its object should not possess it.

Internal: unthinkable that the object lacks it. The two shades of blue.

(This shade of blue and that one stand, eo ipso, in the internal relation of lighter to darker. It is unthinkable that *these* two objects should not stand in this relation.)

(Here the shifting use of the word 'object' corresponds to the shifting use of the words 'property' and 'relation'.)

4.124 The existence of an internal property of a possible situation is not expressed by means of a proposition: rather, it expresses itself in the proposition representing the situation, by means of an internal property of that proposition.

It would be just as nonsensical to assert that a proposition had a formal property as to deny it.

4.1241 It is impossible to distinguish forms from one another by saying that one has this property and another that property: for this presupposes that it makes sense to ascribe either property to either form.

4.125 The existence of an internal relation between possible situations expresses itself in language by means of an internal relation between the propositions representing them.

4.1251 Here we have the answer to the vexed question 'whether all relations are internal or external'.

4.1252 I call a series that is ordered by an *internal* relation a series of forms.

The order of the number-series is not governed by an external relation but by an internal relation.

The same is true of the series of propositions
'aRb',
'$(\exists x):aRx.xRb$',
'$(\exists x, y):aRx.xRy.yRb$',
and so forth.

(If b stands in one of these relations to a, I call b a successor of a.)

4.126 We can now talk about formal concepts, in the same sense that we speak of formal properties.

(I introduce this expression in order to exhibit the source of the confusion between formal concepts and concepts proper, which pervades the whole of traditional logic.)

When something falls under a formal concept as one of its objects, this cannot be expressed by means of a proposition. Instead it is shown in the very sign for this object. (A name shows that it signifies an object, a sign for a number that it signifies a number, etc.)

Formal concepts cannot, in fact, be represented by means of a function, as concepts proper can.

For their characteristics, formal properties, are not expressed by means of functions.

The expression for a formal property is a feature of certain symbols.

So the sign for the characteristics of a formal concept is a distinctive feature of all symbols whose meanings fall under the concept.

So the expression for a formal concept is a propositional variable in which this distinctive feature alone is constant.

4.127 The propositional variable signifies the formal concept, and its values signify the objects that fall under the concept.

4.1271 Every variable is the sign for a formal concept.

For every variable represents a constant form that all its values possess, and this can be regarded as a formal property of those values.

4.1272 Thus the variable name 'x' is the proper sign for the pseudo-concept *object*.

Wherever the word 'object' ('thing', etc.) is correctly used, it is expressed in conceptual notation by a variable name.

For example, in the proposition, 'There are 2 objects which. . .', it is expressed by '$(\exists x, y)$. . .'.

'Object' is a formal concept. So 'There are objects' is nonsense, and so is counting them.

Wherever it is used in a different way, that is as a proper concept-word, nonsensical pseudo-propositions are the result.

So one cannot say, for example, 'There are objects', as one might say, 'There are books'. And it is just as impossible to say, 'There are 100 objects', or, 'There are $\aleph_0$ objects'.

And it is nonsensical to speak of the *total number of objects*.

The same applies to the words 'complex', 'fact', 'function', 'number', etc.

They all signify formal concepts, and are represented in conceptual notation by variables, not by functions or classes (as Frege and Russell believed).

'1 is a number', 'There is only one zero', and all similar expressions are nonsensical.

(It is just as nonsensical to say, 'There is only one 1', as it would be to say, '2+2 at 3 o'clock equals 4'.)

4.12721 A formal concept is given immediately any object falling under it is given. It is not possible, therefore, to introduce as primitive ideas objects belonging to a formal concept and the formal concept itself. So it is impossible, for example, to introduce as primitive ideas both the concept of a function and specific functions, as Russell does; or the concept of a number and particular numbers.

4.1273 If we want to express in conceptual notation the general proposition, 'b is a successor of a', then we require an expression for the general term of the series of forms

$$aRb,$$
$$(\exists x):aRx.xRb,$$
$$(\exists x, y):aRx.xRy.yRb,$$
$$\ldots.$$

In order to express the general term of a series of forms, we must use a variable, because the concept 'term of that series of forms' is a *formal* concept. (This is what Frege and Russell overlooked: consequently the way in which they want to express general propositions like the one above is incorrect; it contains a vicious circle.)

We can determine the general term of a series of forms by giving its first term and the general form of the operation that produces the next term out of the proposition that precedes it.

4.1274 To ask whether a formal concept exists is nonsensical. For no proposition can be the answer to such a question.

(So, for example, the question, 'Are there unanalysable subject-predicate propositions?' cannot be asked.)

4.128 Logical forms are *without* number.

Hence there are no pre-eminent numbers in logic, and hence there is no possibility of philosophical monism or dualism, etc.

Sense: agreement and disagreement with what could exist or not.

4.2 The sense of a proposition is its agreement and disagreement with possibilities of existence and non-existence of states of affairs.

An elementary proposition asserts one state of affairs. No elementary one contradicts it.

4.21 The simplest kind of proposition, an elementary proposition, asserts the existence of a state of affairs.

4.211 It is a sign of a proposition's being elementary that there can be no elementary proposition contradicting it.

Nothing but names in immediate combination, and analysis must end there.

4.22 An elementary proposition consists of names. It is a nexus, a concatenation, of names.

4.221 It is obvious that the analysis of propositions must bring us to elementary propositions which consist of names in immediate combination.

This raises the question how such combination into propositions comes about.

4.2211 Even if the world is infinitely complex, so that every fact consists of infinitely many states of affairs and every state of affairs is composed of infinitely many objects, there would still have to be objects and states of affairs.

4.23 It is only in the nexus of an elementary proposition that a name occurs in a proposition.

4.24 Names are the simple symbols: I indicate them by single letters ('x', 'y', 'z').

I write elementary propositions as functions of names, so that they have the form 'fx', 'φ(x,y)', etc.

Or I indicate them by the letters 'p', 'q', 'r'.

4.241 When I use two signs with one and the same meaning, I express this by putting the sign '=' between them.

So '$a = b$' means that the sign 'b' can be substituted for the sign 'a'.

(If I use an equation to introduce a new sign 'b', laying down that it shall serve as a substitute for a sign 'a' that is already known, then, like Russell, I write the equation—definition—in the form '$a = b$ Def.' A definition is a rule dealing with signs.)

4.242 Expressions of the form '$a = b$' are, therefore, mere representational devices. They state nothing about the meaning of the signs 'a' and 'b'.

4.243 Can we understand two names without knowing whether they signify the same thing or two different things?—Can we understand a proposition in which two names occur without knowing whether their meaning is the same or different?

Suppose I know the meaning of an English word and of a German word that means the same: then it is impossible for me to be unaware that they do mean the same; I must be capable of translating each into the other.

Expressions like '$a = a$', and those derived from them, are neither elementary propositions nor is there any other way in which they have sense. (This will become evident later.)

True if the state of affairs exists, false if it doesn't.

4.25 If an elementary proposition is true, the state of affairs exists: if an elementary proposition is false, the state of affairs does not exist.

All of them with their truth-values: the world completely described.

4.26 If all true elementary propositions are given, the result is a complete description of the world. The world is completely described by giving all elementary propositions, and adding which of them are true and which false.

4.27 For n states of affairs, there are $K_n = \sum_{v=0}^{n} \binom{n}{v}$ possibilities of existence and non-existence.

Of these states of affairs any combination can exist and the remainder not exist.

4.28 There correspond to these combinations the same number of possibilities of truth—and falsity—for n elementary propositions.

Truth-possibilities: which states of affairs exist and which don't. The rows of Ts and Fs set them out.

4.3 Truth-possibilities of elementary propositions mean possibilities of existence and non-existence of states of affairs.

4.31 We can represent truth-possibilities by schemata of the following kind ('T' means 'true', 'F' means 'false'; the rows of 'T's' and 'F's' under the row of elementary propositions symbolize their truth-possibilities in a way that can easily be understood):

p	q	r
T	T	T
F	T	T
T	F	T
T	T	F
F	F	T
F	T	F
T	F	F
F	F	F

,

p	q
T	T
F	T
T	F
F	F

,

p
T
F

.

A proposition agrees with some truth-possibilities and disagrees with the rest.

4.4 A proposition is an expression of agreement and disagreement with truth-possibilities of elementary propositions.

4.41 Truth-possibilities of elementary propositions are the conditions of the truth and falsity of propositions.

4.411 It immediately strikes one as probable that the introduction of elementary propositions provides the basis for understanding all other kinds of proposition. Indeed the understanding of general propositions *palpably* depends on the understanding of elementary propositions.

4.42 For n elementary propositions there are $\sum_{k=0}^{K_n} \binom{K_n}{k} = L_n$ ways in which a proposition can agree and disagree with their truth-possibilities

4.43 We can express agreement with truth-possibilities by correlating the mark 'T' (true) with them in the schema.

The absence of this mark means disagreement.

4.431 The expression of agreement and disagreement with the truth-possibilities of elementary propositions expresses the truth-conditions of a proposition.

Sense as truth-conditions, which is where Frege rightly started.

A proposition is the expression of its truth-conditions.

(Thus Frege was quite right to use them as a starting point when he explained the signs of his conceptual notation. But the explanation of the concept of truth that Frege gives is mistaken: if 'the true' and 'the false' were really objects, and were the arguments in ~p etc., then Frege's method of determining the sense of '~p' would leave it absolutely undetermined.)

4.44 The sign that results from correlating the mark 'T' with truth-possibilities is a propositional sign.

Nothing answers to the Ts and Fs, or to the lines and brackets. No logical objects.

4.441 It is clear that a complex of the signs 'F' and 'T' has no object (or complex of objects) corresponding to it, just as there is none corresponding to the horizontal and vertical lines or to the brackets.—There are no 'logical objects'.

The truth-table as a propositional sign.

Of course the same applies to all signs that express what the schemata of 'T's' and 'F's' express.

4.442 For example, the following is a propositional sign:

'p	q	'
T	T	T
F	T	T
T	F	
F	F	T.

(Frege's 'judgement-stroke' '⊢' is logically quite meaningless: in the works of Frege (and Russell) it simply indicates that these authors hold the propositions marked with this sign to be true. Thus '⊢' is no more a component part of a proposition than is, for instance, the proposition's number. It is quite impossible for a proposition to state that it itself is true.)

The last column written as a row: a sign with no connective in it.

If the order of the truth-possibilities in a schema is fixed once and for all by a combinatory rule, then the last column by itself will be an expression of the truth-conditions. If we now write this column as a row, the propositional sign will become

'(TT-T) (p,q)'

or more explicitly

'(TTFT) (p,q)'.

(The number of places in the left-hand pair of brackets is determined by the number of terms in the right-hand pair.)

4.45 For n elementary propositions there are L_n possible groups of truth-conditions.

The groups of truth-conditions that are obtainable

from the truth-possibilities of a given number of elementary propositions can be arranged in a series.

4.46 Among the possible groups of truth-conditions there are two extreme cases.

The two extreme cases: true for every truth-possibility, or for none.

In one of these cases the proposition is true for all the truth-possibilities of the elementary propositions. We say that the truth-conditions are *tautological*.

In the second case the proposition is false for all the truth-possibilities: the truth-conditions are *contradictory*.

In the first case we call the proposition a tautology; in the second, a contradiction.

4.461 Propositions show what they say: tautologies and contradictions show that they say nothing.

No truth-conditions, so a tautology says nothing. It lacks sense.

A tautology has no truth-conditions, since it is unconditionally true: and a contradiction is true on no condition.

Tautologies and contradictions lack sense.

(Like a point from which two arrows go out in opposite directions to one another.)

The weather example. Senseless isn't nonsensical: they belong to the symbolism, as '0' does.

(For example, I know nothing about the weather when I know that it is either raining or not raining.)

4.4611 Tautologies and contradictions are not, however, nonsensical. They are part of the symbolism, much as '0' is part of the symbolism of arithmetic.

4.462 Tautologies and contradictions are not pictures of reality. They do not represent any possible situations. For the former admit *all* possible situations, and the latter *none*.

Not pictures: the representational relations cancel one another.

In a tautology the conditions of agreement with the world—the representational relations—cancel one another, so that it does not stand in any representational relation to reality.

4.463 The truth-conditions of a proposition determine the range that it leaves open to the facts.

(A proposition, a picture, or a model is, in the negative sense, like a solid body that restricts the freedom of movement of others, and, in the positive sense, like a space bounded by solid substance in which there is room for a body.)

All of logical space left open, or all of it filled. Neither determines reality.

A tautology leaves open to reality the whole—the infinite whole—of logical space: a contradiction fills the whole of logical space leaving no point of it for reality. Thus neither of them can determine reality in any way.

4.464 A tautology's truth is certain, a proposition's possible, a contradiction's impossible.

(Certain, possible, impossible: here we have the first indication of the scale that we need in the theory of probability.)

Conjoined with a tautology, a proposition says just what it said before.

4.465 The logical product of a tautology and a proposition says the same thing as the proposition. This product, therefore, is identical with the proposition. For it is impossible to alter what is essential to a symbol without altering its sense.

4.466 What corresponds to a determinate logical combination of signs is a determinate logical combination of their meanings. It is only to the uncombined signs that *absolutely any* combination corresponds.

In other words, propositions that are true for every situation cannot be combinations of signs at all, since, if they were, only determinate combinations of objects could correspond to them.

(And what is not a logical combination has *no* combination of objects corresponding to it.)

Limiting cases, where the combination of signs comes apart.

Tautology and contradiction are the limiting cases—indeed the disintegration—of the combination of signs.

4.4661 Admittedly the signs are still combined with one another even in tautologies and contradictions—i.e. they

stand in certain relations to one another: but these relations have no meaning, they are not essential to the *symbol*.

4.5 It now seems possible to give the most general propositional form: that is, to give a description of the propositions of *any* sign-language *whatsoever* in such a way that every possible sense can be expressed by a symbol satisfying the description, and every symbol satisfying the description can express a sense, provided that the meanings of the names are suitably chosen.

It is clear that *only* what is essential to the most general propositional form may be included in its description—for otherwise it would not be the most general form.

The existence of a general propositional form is proved by the fact that there cannot be a proposition whose form could not have been foreseen (i.e. constructed). The general form of a proposition is: This is how things stand.

4.51 Suppose that I am given *all* elementary propositions: then I can simply ask what propositions I can construct out of them. And there I have *all* propositions, and *that* fixes their limits.

4.52 Propositions comprise all that follows from the totality of all elementary propositions (and, of course, from its being the *totality* of them *all*). (Thus, in a certain sense, it could be said that *all* propositions were generalizations of elementary propositions.)

4.53 The general propositional form is a variable.

5 A proposition is a truth-function of elementary propositions.

Proposition 5: every proposition is a truth-function of elementary ones.

(An elementary proposition is a truth-function of itself.)

5.01 Elementary propositions are the truth-arguments of propositions.

5.02 The arguments of functions are readily confused with the affixes of names. For both arguments and affixes enable me to recognize the meaning of the signs containing them.

For example, when Russell writes '$+_c$', the '$_c$' is an affix which indicates that the sign as a whole is the addition-sign for cardinal numbers. But the use of this sign is the result of arbitrary convention and it would be quite possible to choose a simple sign instead of '$+_c$'; in '$\sim p$', however, 'p' is not an affix but an argument: the sense of '$\sim p$' *cannot* be understood unless the sense of 'p' has been understood already. (In the name Julius Caesar 'Julius' is an affix. An affix is always part of a description of the object to whose name we attach it: e.g. *the* Caesar of the Julian gens.)

If I am not mistaken, Frege's theory about the meaning of propositions and functions is based on the confusion between an argument and an affix. Frege regarded the propositions of logic as names, and their arguments as the affixes of those names.

5.1 Truth-functions can be arranged in series.

That is the foundation of the theory of probability.

5.101 The truth-functions of a given number of elementary propositions can always be set out in a schema of the following kind:

(T T T T) (p, q) Tautology (If p then p, and if q then q.) ($p \supset p \,.\, q \supset q$)
(F T T T) (p, q) In words: Not both p and q. ($\sim(p \,.\, q)$)
(T F T T) (p, q) ,, ,, : If q then p. ($q \supset p$)
(T T F T) (p, q) ,, ,, : If p then q. ($p \supset q$)
(T T T F) (p, q) ,, ,, : p or q. ($p \vee q$)
(F F T T) (p, q) ,, ,, : Not q. ($\sim q$)
(F T F T) (p, q) ,, ,, : Not p. ($\sim p$)
(F T T F) (p, q) ,, ,, : p or q, but not both. ($p \,.\, \sim q : v : q \,.\, \sim p$)
(T F F T) (p, q) ,, ,, : If p then q, and if q then p. ($p \equiv q$)
(T F T F) (p, q) ,, ,, : p

(T T F F) (p, q) ,, ,, : q
(F F F T) (p, q) ,, ,, : Neither p nor q. ($\sim p \,.\, \sim q$ or $p \mid q$)
(F F T F) (p, q) ,, ,, : p and not q. ($p \,.\, \sim q$)
(F T F F) (p, q) ,, ,, : q and not p. ($q \,.\, \sim p$)
(T F F F) (p, q) ,, ,, : q and p. ($q \,.\, p$)
(F F F F) (p, q) Contradiction (p and not p, and q and not q.) ($p \,.\, \sim p \,.\, q \,.\, \sim q$)

I will give the name truth-grounds of a proposition to those truth-possibilities of its truth-arguments that make it true.

5.11 If all the truth-grounds that are common to a number of propositions are at the same time truth-grounds of a certain proposition, then we say that the truth of that proposition follows from the truth of the others.

5.12 In particular, the truth of a proposition 'p' follows from the truth of another proposition 'q' if all the truth-grounds of the latter are truth-grounds of the former.

5.121 The truth-grounds of the one are contained in those of the other: p follows from q.

5.122 If p follows from q, the sense of 'p' is contained in the sense of 'q'.

5.123 If a god creates a world in which certain propositions are true, then by that very act he also creates a world in which all the propositions that follow from them come true. And similarly he could not create a world in which the proposition 'p' was true without creating all its objects.

5.124 A proposition affirms every proposition that follows from it.

5.1241 '$p \,.\, q$' is one of the propositions that affirm 'p' and at the same time one of the propositions that affirm 'q'.

Two propositions are opposed to one another if there is no proposition with a sense, that affirms them both.

Every proposition that contradicts another negates it.

5.13 When the truth of one proposition follows from the truth of others, we can see this from the structure of the propositions.

5.131 If the truth of one proposition follows from the truth of others, this finds expression in relations in which the forms of the propositions stand to one another: nor is it necessary for us to set up these relations between them, by combining them with one another in a single proposition; on the contrary, the relations are internal, and their existence is an immediate result of the existence of the propositions.

5.1311 When we infer q from p v q and ~p, the relation between the propositional forms of 'p v q' and '~p' is masked, in this case, by our mode of signifying. But if instead of 'p v q' we write, for example, 'p|q.|.p|q', and instead of '~p', 'p|p' (p|q = neither p nor q), then the inner connexion becomes obvious.

(The possibility of inference from (x).fx to fa shows that the symbol (x).fx itself has generality in it.)

5.132 If p follows from q, I can make an inference from q to p, deduce p from q.

The nature of the inference can be gathered only from the two propositions.

They themselves are the only possible justification of the inference.

'Laws of inference', which are supposed to justify inferences, as in the works of Frege and Russell, have no sense, and would be superfluous.

5.133 All deductions are made a priori.

5.134 One elementary proposition cannot be deduced from another.

5.135 There is no possible way of making an inference from the existence of one situation to the existence of another, entirely different situation.

Independence again: nothing can be inferred from one situation about an entirely different one.

5.136 There is no causal nexus to justify such an inference.

5.1361 We *cannot* infer the events of the future from those of the present.

Superstition is nothing but belief in the causal nexus.

5.1362 The freedom of the will consists in the impossibility of knowing actions that still lie in the future. We could know them only if causality were an *inner* necessity like that of logical inference.—The connexion between knowledge and what is known is that of logical necessity.

('*A* knows that p is the case', has no sense if p is a tautology.)

5.1363 If the truth of a proposition does not *follow* from the fact that it is self-evident to us, then its self-evidence in no way justifies our belief in its truth.

5.14 If one proposition follows from another, then the latter says more than the former, and the former less than the latter.

5.141 If p follows from q and q from p, then they are one and the same proposition.

5.142 A tautology follows from all propositions: it says nothing.

5.143 Contradiction is that common factor of propositions which *no* proposition has in common with another. Tautology is the common factor of all propositions that have nothing in common with one another.

Contradiction, one might say, vanishes outside all propositions: tautology vanishes inside them.

Contradiction is the outer limit of propositions: tautology is the unsubstantial point at their centre.

5.15 If T_r is the number of the truth-grounds of a proposition 'r', and if T_{rs} is the number of the truth-grounds of a proposition '*s*' that are at the same time truth-grounds of 'r', then we call the ratio T_{rs}: T_r the degree of *probability* that the proposition 'r' gives to the proposition '*s*'.

5.151 In a schema like the one above in 5.101, let T_r be the number of 'T's' in the proposition r, and let T_{rs} be the number of 'T's' in the proposition *s* that stand in columns in which the proposition r has 'T's'. Then the proposition r gives to the proposition *s* the probability T_{rs}: T_r.

5.1511 There is no special object peculiar to probability propositions.

5.152 When propositions have no truth-arguments in common with one another, we call them independent of one another.

Two elementary propositions give one another the probability $\frac{1}{2}$.

If p follows from q, then the proposition 'q' gives to the proposition 'p' the probability 1. The certainty of logical inference is a limiting case of probability.

(Application of this to tautology and contradiction.)

5.153 In itself, a proposition is neither probable nor improbable. Either an event occurs or it does not: there is no middle way.

5.154 Suppose that an urn contains black and white balls in equal numbers (and none of any other kind). I draw one ball after another, putting them back into the urn. By this experiment I can establish that the number of black balls drawn and the number of white balls drawn approximate to one another as the draw continues.

So *this* is not a mathematical truth.

Now, if I say, 'The probability of my drawing a white ball is equal to the probability of my drawing a black one', this means that all the circumstances that I know of (including the laws of nature assumed as hypotheses) give no *more* probability to the occurrence of the one event than to that of the other. That is to say, they give each the probability $\frac{1}{2}$, as can easily be gathered from the above definitions.

What I confirm by the experiment is that the occurrence of the two events is independent of the circumstances of which I have no more detailed knowledge.

5.155 The minimal unit for a probability proposition is this: The circumstances—of which I have no further knowledge—give such and such a degree of probability to the occurrence of a particular event.

5.156 It is in this way that probability is a generalization.

It involves a general description of a propositional form.

We use probability only in default of certainty—if our knowledge of a fact is not indeed complete, but we do know *something* about its form.

(A proposition may well be an incomplete picture of a certain situation, but it is always a complete picture of *something*.)

A probability proposition is a sort of excerpt from other propositions.

5.2 The structures of propositions stand in internal relations to one another.

5.21 In order to give prominence to these internal relations we can adopt the following mode of expression: we can represent a proposition as the result of an operation that

produces it out of other propositions (which are the bases of the operation).

5.22 An operation is the expression of a relation between the structures of its result and of its bases.

5.23 The operation is what has to be done to the one proposition in order to make the other out of it.

5.231 And that will, of course, depend on their formal properties, on the internal similarity of their forms.

5.232 The internal relation by which a series is ordered is equivalent to the operation that produces one term from another.

5.233 Operations cannot make their appearance before the point at which one proposition is generated out of another in a logically meaningful way; i.e. the point at which the logical construction of propositions begins.

5.234 Truth-functions of elementary propositions are results of operations with elementary propositions as bases. (These operations I call truth-operations.)

5.2341 The sense of a truth-function of p is a function of the sense of p.

Negation, logical addition, logical multiplication, etc. etc. are operations.

(Negation reverses the sense of a proposition.)

5.24 An operation manifests itself in a variable; it shows how we can get from one form of proposition to another.

It gives expression to the difference between the forms.

(And what the bases of an operation and its result have in common is just the bases themselves.)

5.241 An operation is not the mark of a form, but only of a difference between forms.

5.242 The operation that produces 'q' from 'p' also produces 'r' from 'q', and so on. There is only one way of expressing this: 'p', 'q', 'r', etc. have to be variables that give expression in a general way to certain formal relations.

5.25 The occurrence of an operation does not characterize the sense of a proposition.

Indeed, no statement is made by an operation, but only by its result, and this depends on the bases of the operation.

(Operations and functions must not be confused with each other.)

5.251 A function cannot be its own argument, whereas an operation can take one of its own results as its base.

5.252 It is only in this way that the step from one term of a series of forms to another is possible (from one type to another in the hierarchies of Russell and Whitehead).

(Russell and Whitehead did not admit the possibility of such steps, but repeatedly availed themselves of it.)

5.2521 If an operation is applied repeatedly to its own results, I speak of successive applications of it. ('$O'O'O'a$' is the result of three successive applications of the operation '$O'\xi$' to 'a'.)

In a similar sense I speak of successive applications of *more than one* operation to a number of propositions.

5.2522 Accordingly I use the sign '$[a, x, O'x]$' for the general term of the series of forms a, $O'a$, $O'O'a$, ...This bracketed expression is a variable: the first term of the bracketed expression is the beginning of the series of forms, the second is the form of a term x arbitrarily selected from the series, and the third is the form of the term that immediately follows x in the series.

5.2523 The concept of successive applications of an operation is equivalent to the concept 'and so on'.

Operations can cancel and vanish, as the two negations in not-not-p do.

5.253 One operation can counteract the effect of another. Operations can cancel one another.

5.254 An operation can vanish (e.g. negation in '~~p': ~~p = p).

5.3 All propositions are results of truth-operations on elementary propositions.

A truth-operation is the way in which a truth-function is produced out of elementary propositions.

It is of the essence of truth-operations that, just as elementary propositions yield a truth-function of themselves, so too in the same way truth-functions yield a further truth-function. When a truth-operation is applied to truth-functions of elementary propositions, it always generates another truth-function of elementary propositions, another proposition. When a truth-operation is applied to the results of truth-operations on elementary propositions, there is always a *single* operation on elementary propositions that has the same result.

Every proposition is the result of truth-operations on elementary propositions.

5.31 The schemata in 4.31 have a meaning even when 'p', 'q', 'r', etc. are not elementary propositions.

And it is easy to see that the propositional sign in 4.442 expresses a single truth-function of elementary propositions even when 'p' and 'q' are truth-functions of elementary propositions.

5.32 All truth-functions are results of successive applications to elementary propositions of a finite number of truth-operations.

5.4 At this point it becomes manifest that there are no 'logical objects' or 'logical constants' (in Frege's and Russell's sense).

No logical objects, no logical constants in Frege's and Russell's sense.

5.41 The reason is that the results of truth-operations on truth-functions are always identical whenever they are one and the same truth-function of elementary propositions.

5.42 It is self-evident that v, ⊃, etc. are not relations in the sense in which right and left etc. are relations.

The interdefinability of Frege's and Russell's 'primitive signs' of logic is enough to show that they are not primitive signs, still less signs for relations.

The signs can be defined by one another, so they aren't primitive, still less signs for relations.

And it is obvious that the '⊃' defined by means of '~' and 'v' is identical with the one that figures with '~' in the definition of 'v'; and that the second 'v' is identical with the first one; and so on.

5.43 Even at first sight it seems scarcely credible that there should follow from one fact p infinitely many others, namely ~~p, ~~~~p, etc. And it is no less remarkable that the infinite number of propositions of logic (mathematics) follow from half a dozen 'primitive propositions'.

But in fact all the propositions of logic say the same thing, to wit nothing.

The propositions of logic all say the same thing: nothing.

5.44 Truth-functions are not material functions.

For example, an affirmation can be produced by double negation: in such a case does it follow that in some sense negation is contained in affirmation? Does '~~p' negate ~p, or does it affirm p—or both?

Not-not-p isn't about negation. If negation were an object, it would say something different from p.

The proposition '~~p' is not about negation, as if negation were an object: on the other hand, the possibility of negation is already written into affirmation.

And if there were an object called '~', it would follow that '~~p' said something different from what 'p' said,

just because the one proposition would then be about ~ and the other would not.

5.441 This vanishing of the apparent logical constants also occurs in the case of '$\sim(\exists x).\sim fx$', which says the same as '$(x).fx$', and in the case of '$(\exists x).fx.x = a$', which says the same as 'fa'.

5.442 If we are given a proposition, then with it we are also given the results of all truth-operations that have it as their base.

5.45 If there are primitive logical signs, then any logic that fails to show clearly how they are placed relatively to one another and to justify their existence will be incorrect. The construction of logic out of its primitive signs must be made clear.

5.451 If logic has primitive ideas, they must be independent of one another. If a primitive idea has been introduced, it must have been introduced in all the combinations in which it ever occurs. It cannot, therefore, be introduced first for *one* combination and later re-introduced for another. For example, once negation has been introduced, we must understand it both in propositions of the form '$\sim p$' and in propositions like '$\sim(p \vee q)$', '$(\exists x).\sim fx$', etc. We must not introduce it first for the one class of cases and then for the other, since it would then be left in doubt whether its meaning were the same in both cases, and no reason would have been given for combining the signs in the same way in both cases.

(In short, Frege's remarks about introducing signs by means of definitions (in *The Fundamental Laws of Arithmetic*) also apply, mutatis mutandis, to the introduction of primitive signs.)

5.452 The introduction of any new device into the symbolism

of logic is necessarily a momentous event. In logic a new device should not be introduced in brackets or in a footnote with what one might call a completely innocent air.

(Thus in Russell and Whitehead's *Principia Mathematica* there occur definitions and primitive propositions expressed in words. Why this sudden appearance of words? It would require a justification, but none is given, or could be given, since the procedure is in fact illicit.)

But if the introduction of a new device has proved necessary at a certain point, we must immediately ask ourselves, 'At what points is the employment of this device now *unavoidable*?' and its place in logic must be made clear.

5.453 All numbers in logic stand in need of justification.

Or rather, it must become evident that there are no numbers in logic.

There are no pre-eminent numbers.

5.454 In logic there is no co-ordinate status, and there can be no classification.

In logic there can be no distinction between the general and the specific.

5.4541 The solutions of the problems of logic must be simple, since they set the standard of simplicity.

Men have always had a presentiment that there must be a realm in which the answers to questions are symmetrically combined—a priori—to form a self-contained system.

A realm subject to the law: Simplex sigillum veri.

5.46 If we introduced logical signs properly, then we should also have introduced at the same time the sense of all combinations of them; i.e. not only '$p \vee q$' but '$\sim(p \vee \sim q)$'

as well, etc. etc. We should also have introduced at the same time the effect of all possible combinations of brackets. And thus it would have been made clear that the real general primitive signs are not 'p v q', '$(\exists x).fx$', etc. but the most general form of their combinations.

5.461 Though it seems unimportant, it is in fact significant that the pseudo-relations of logic, such as v and $\supset$, need brackets—unlike real relations.

Indeed, the use of brackets with these apparently primitive signs is itself an indication that they are not the real primitive signs. And surely no one is going to believe that brackets have an independent meaning.

5.4611 Signs for logical operations are punctuation-marks.

5.47 It is clear that whatever we can say in *advance* about the form of all propositions, we must be able to say *all at once*.

An elementary proposition really contains all logical operations in itself. For 'fa' says the same thing as

$$'(\exists x).fx.x = a'.$$

Wherever there is compositeness, argument and function are present, and where these are present, we already have all the logical constants.

One could say that the sole logical constant was what *all* propositions, by their very nature, had in common with one another.

But that is the general propositional form.

5.471 The general propositional form is the essence of a proposition.

5.4711 To give the essence of a proposition means to give the essence of all description, and thus the essence of the world.

5.472 The description of the most general propositional form

is the description of the one and only general primitive sign in logic.

5.473 Logic must look after itself.

If a sign is *possible*, then it is also capable of signifying. Whatever is possible in logic is also permitted. (The reason why 'Socrates is identical' means nothing is that there is no property called 'identical'. The proposition is nonsensical because we have failed to make an arbitrary determination, and not because the symbol, in itself, would be illegitimate.)

'Socrates is identical': nonsense because we've fixed no meaning, not because the symbol is illegitimate.

In a certain sense, we cannot make mistakes in logic.

5.4731 Self-evidence, which Russell talked about so much, can become dispensable in logic, only because language itself prevents every logical mistake.—What makes logic a priori is the *impossibility* of illogical thought.

5.4732 We cannot give a sign the wrong sense.

5.47321 Occam's maxim is, of course, not an arbitrary rule, nor one that is justified by its success in practice: its point is that *unnecessary* units in a sign-language mean nothing.

Signs that serve *one* purpose are logically equivalent, and signs that serve *none* are logically meaningless.

5.4733 Frege says that any legitimately constructed proposition must have a sense. And I say that any possible proposition is legitimately constructed, and, if it has no sense, that can only be because we have failed to give a *meaning* to some of its constituents.

Any possible proposition is legitimately constructed. If one lacks sense, we've failed to give some part of it a meaning.

(Even if we think that we have done so.)

Thus the reason why 'Socrates is identical' says nothing is that we have not given *any adjectival* meaning to the word 'identical'. For when it appears as a sign for identity, it symbolizes in an entirely different way—the signifying relation is a different one—therefore the

symbols also are entirely different in the two cases: the two symbols have only the sign in common, and that is an accident.

5.474 The number of fundamental operations that are necessary depends *solely* on our notation.

5.475 All that is required is that we should construct a system of signs with a particular number of dimensions—with a particular mathematical multiplicity.

5.476 It is clear that this is not a question of a *number of primitive ideas* that have to be signified, but rather of the expression of a rule.

5.5 Every truth-function is a result of successive applications to elementary propositions of the operation

$$\text{`(-----T)}(\xi, \ldots.)\text{'}.$$

This operation negates all the propositions in the right-hand pair of brackets, and I call it the negation of those propositions.

5.501 When a bracketed expression has propositions as its terms—and the order of the terms inside the brackets is indifferent—then I indicate it by a sign of the form '$(\bar{\xi})$'. 'ξ' is a variable whose values are terms of the bracketed expression and the bar over the variable indicates that it is the representative of all its values in the brackets.

(E.g. if ξ has the three values P, Q, R, then

$$(\bar{\xi}) = (P, Q, R).)$$

What the values of the variable are is something that is stipulated.

The stipulation is a description of the propositions that have the variable as their representative.

How the description of the terms of the bracketed expression is produced is not essential.

We *can* distinguish three kinds of description: 1. direct enumeration, in which case we can simply substitute for the variable the constants that are its values; 2. giving a function *fx* whose values for all values of *x* are the propositions to be described; 3. giving a formal law that governs the construction of the propositions, in which case the bracketed expression has as its members all the terms of a series of forms.

5.502 So instead of '$\text{(-----T)}(\xi, \ldots.)$', I write '$N(\bar{\xi})$'.

$N(\bar{\xi})$ is the negation of all the values of the propositional variable $\bar{\xi}$.

5.503 It is obvious that we can easily express how propositions may be constructed with this operation, and how they may not be constructed with it; so it must be possible to find an exact expression for this.

5.51 If $\bar{\xi}$ has only one value, then $N(\bar{\xi}) = \sim p$ (not p); if it has two values, then $N(\bar{\xi}) = \sim p.\sim q$ (neither p nor q).

5.511 How can logic—all-embracing logic, which mirrors the world—use such peculiar crotchets and contrivances? Only because they are all connected with one another in an infinitely fine network, the great mirror.

5.512 '$\sim p$' is true if 'p' is false. Therefore, in the proposition '$\sim p$', when it is true, 'p' is a false proposition. How then can the stroke '$\sim$' make it agree with reality?

But in '$\sim p$' it is not '$\sim$' that negates; it is rather what is common to all the signs of this notation that negate p.

That is to say the common rule that governs the construction of '$\sim p$', '$\sim\sim\sim p$', '$\sim p \vee \sim p$', '$\sim p.\sim p$', etc. etc. (ad inf.). And this common factor mirrors negation.

5.513 We might say that what is common to all symbols that

affirm both p and q is the proposition 'p.q'; and that what is common to all symbols that affirm either p or q is the proposition 'p v q'.

And similarly we can say that two propositions are opposed to one another if they have nothing in common with one another, and that every proposition has only one negative, since there is only one proposition that lies completely outside it.

Conjoin 'p or not p' with q and the result still says just what q says.

Thus in Russell's notation too it is manifest that 'q:p v ~p' says the same thing as 'q', that 'p v ~p' says nothing.

5.514 Once a notation has been established, there will be in it a rule governing the construction of all propositions that negate p, a rule governing the construction of all propositions that affirm p, and a rule governing the construction of all propositions that affirm p or q; and so on. These rules are equivalent to the symbols; and in them their sense is mirrored.

5.515 It must be manifest in our symbols that it can only be propositions that are combined with one another by 'v', '.', etc.

And this is indeed the case, since the symbol in 'p' and 'q' itself presupposes 'v', '~', etc. If the sign 'p' in 'p v q' does not stand for a complex sign, then it cannot have sense by itself: but in that case the signs 'p v p', 'p.p', etc., which have the same sense as p, must also lack sense. But if 'p v p' has no sense, then 'p v q' cannot have a sense either.

5.5151 Must the sign of a negative proposition be constructed with that of the positive proposition? Why should it not be possible to express a negative proposition by means of a negative fact? (E.g. suppose that 'a' does not stand in a certain relation to 'b'; then this might be used to say that *aRb* was not the case.)

But really even in this case the negative proposition is constructed by an indirect use of the positive.

The positive *proposition* necessarily presupposes the existence of the negative *proposition* and vice versa.

5.52 If ξ has as its values all the values of a function fx for all values of x, then $N(\xi) = \sim(\exists x).fx$.

5.521 I dissociate the concept *all* from truth-functions.

Frege and Russell introduced generality in association with logical product or logical sum. This made it difficult to understand the propositions '$(\exists x).fx$' and '$(x).fx$', in which both ideas are embedded.

5.522 What is peculiar to the generality-sign is first, that it indicates a logical prototype, and secondly, that it gives prominence to constants.

5.523 The generality-sign occurs as an argument.

5.524 If objects are given, then at the same time we are given *all* objects.

If elementary propositions are given, then at the same time *all* elementary propositions are given.

5.525 It is incorrect to render the proposition '$(\exists x).fx$' in the words, 'fx is *possible*', as Russell does.

The certainty, possibility, or impossibility of a situation is not expressed by a proposition, but by an expression's being a tautology, a proposition with sense, or a contradiction.

The precedent to which we are constantly inclined to appeal must reside in the symbol itself.

5.526 We can describe the world completely by means of fully generalized propositions, i.e. without first correlating any name with a particular object.

Then, in order to arrive at the customary mode of

expression, we simply need to add, after an expression like, 'There is one and only one x such that . . .', the words, 'and that x is a'.

5.5261 A fully generalized proposition, like every other proposition, is composite. (This is shown by the fact that in '$(\exists x, \varphi).\varphi x$' we have to mention '$\varphi$' and '$x$' separately. They both, independently, stand in signifying relations to the world, just as is the case in ungeneralized propositions.)

It is a mark of a composite symbol that it has something in common with *other* symbols.

5.5262 The truth or falsity of *every* proposition does make some alteration in the general construction of the world. And the range that the totality of elementary propositions leaves open for its construction is exactly the same as that which is delimited by entirely general propositions.

(If an elementary proposition is true, that means, at any rate, *one more* true elementary proposition.)

5.53 Identity of object I express by identity of sign, and not by using a sign for identity. Difference of objects I express by difference of signs.

5.5301 It is self-evident that identity is not a relation between objects. This becomes very clear if one considers, for example, the proposition '$(x):fx.\supset.x = a$'. What this proposition says is simply that *only* a satisfies the function f, and not that only things that have a certain relation to a satisfy the function f.

Of course, it might then be said that *only* a did have this relation to a; but in order to express that, we should need the identity-sign itself.

5.5302 Russell's definition of '=' is inadequate, because according to it we cannot say that two objects have all their properties in common. (Even if this proposition is never correct, it still has *sense*.)

5.5303 Roughly speaking, to say of *two* things that they are identical is nonsense, and to say of *one* thing that it is identical with itself is to say nothing at all.

5.531 Thus I do not write '$f(a,b).a = b$', but '$f(a,a)$' (or '$f(b,b)$'); and not $f(a,b).\sim a = b$', but '$f(a,b)$'.

5.532 And analogously I do not write '$(\exists x,y).f(x,y).x = y$', but '$(\exists x).f(x,x)$'; and not '$(\exists x,y).f(x,y).\sim x = y$', but '$(\exists x.y).f(x,y)$'.

(So Russell's '$(\exists x,y).fxy$' becomes

'$(\exists x.y).f(x,y).v.(\exists x).f(x,x)$'.)

5.5321 Thus, for example, instead of '$(x):fx \supset x = a$' we write '$(\exists x).fx.\supset.fa:\sim(\exists x,y).fx.fy$'.

And the proposition, '*Only one* x satisfies f( )', will read '$(\exists x).fx:\sim(\exists x,y).fx.fy$'.

5.533 The identity-sign, therefore, is not an essential constituent of conceptual notation.

5.534 And now we see that in a correct conceptual notation pseudo-propositions like '$a = a$', '$a = b.b = c.\supset a = c$', '$(x).x = x$', '$(\exists x).x = a$', etc. cannot even be written down.

5.535 This also disposes of all the problems that were connected with such pseudo-propositions.

All the problems that Russell's 'axiom of infinity' brings with it can be solved at this point.

What the axiom of infinity is intended to say would express itself in language through the existence of infinitely many names with different meanings.

5.5351 There are certain cases in which one is tempted to use expressions of the form '$a = a$' or '$p \supset p$' and the like. In fact, this happens when one wants to talk about prototypes, e.g. about proposition, thing, etc. Thus in Russell's

Principles of Mathematics 'p is a proposition'—which is nonsense—was given the symbolic rendering '$p \supset p$' and placed as an hypothesis in front of certain propositions in order to exclude from their argument-places everything but propositions.

(It is nonsense to place the hypothesis '$p \supset p$' in front of a proposition, in order to ensure that its arguments shall have the right form, if only because with a non-proposition as argument the hypothesis becomes not false but nonsensical, and because arguments of the wrong kind make the proposition itself nonsensical, so that it preserves itself from wrong arguments just as well, or as badly, as the hypothesis without sense that was appended for that purpose.)

5.5352 In the same way people have wanted to express, 'There are no *things*', by writing '$\sim(\exists x).x = x$'. But even if this were a proposition, would it not be equally true if in fact 'there were things' but they were not identical with themselves?

5.54 In the general propositional form propositions occur in other propositions only as bases of truth-operations.

5.541 At first sight it looks as if it were also possible for one proposition to occur in another in a different way.

Particularly with certain forms of proposition in psychology, such as '*A* believes that p is the case' and '*A* has the thought p', etc.

For if these are considered superficially, it looks as if the proposition p stood in some kind of relation to an object *A*.

(And in modern theory of knowledge (Russell, Moore, etc.) these propositions have actually been construed in this way.)

5.542 It is clear, however, that '*A* believes that p', '*A* has the

thought p', and '*A* says p' are of the form '"p" says p': and this does not involve a correlation of a fact with an object, but rather the correlation of facts by means of the correlation of their objects.

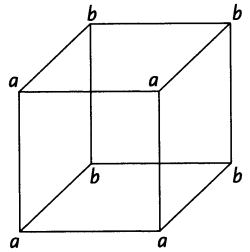
5.5421 This shows too that there is no such thing as the soul—the subject, etc.—as it is conceived in the superficial psychology of the present day.

Indeed a composite soul would no longer be a soul.

5.5422 The correct explanation of the form of the proposition, '*A* makes the judgement p', must show that it is impossible for a judgement to be a piece of nonsense. (Russell's theory does not satisfy this requirement.)

5.5423 To perceive a complex means to perceive that its constituents are related to one another in such and such a way.

This no doubt also explains why there are two possible ways of seeing the figure



as a cube; and all similar phenomena. For we really see two different facts.

(If I look in the first place at the corners marked *a* and only glance at the *b*'s, then the *a*'s appear to be in front, and vice versa).

5.55 We now have to answer a priori the question about all the possible forms of elementary propositions.

Names again, but no telling in advance how elementary propositions are composed.

Elementary propositions consist of names. Since, however, we are unable to give the number of names with different meanings, we are also unable to give the composition of elementary propositions.

5.551 Our fundamental principle is that whenever a question can be decided by logic at all it must be possible to decide it without more ado.

(And if we get into a position where we have to look at the world for an answer to such a problem, that shows that we are on a completely wrong track.)

5.552 The 'experience' that we need in order to understand logic is not that something or other is the state of things, but that something *is*: that, however, is *not* an experience.

Logic *is prior* to every experience—that something *is so*.

It is prior to the question 'How?', not prior to the question 'What?'

5.5521 And if this were not so, how could we apply logic? We might put it in this way: if there would be a logic even if there were no world, how then could there be a logic given that there is a world?

5.553 Russell said that there were simple relations between different numbers of things (individuals). But between what numbers? And how is this supposed to be decided?—By experience?

(There is no pre-eminent number.)

5.554 It would be completely arbitrary to give any specific form.

5.5541 It is supposed to be possible to answer a priori the question whether I can get into a position in which I need the sign for a 27-termed relation in order to signify something.

5.5542 But is it really legitimate even to ask such a question?

Can we set up a form of sign without knowing whether anything can correspond to it?

Does it make sense to ask what there must *be* in order that something can be the case?

5.555 Clearly we have some concept of elementary propositions quite apart from their particular logical forms.

But when there is a system by which we can create symbols, the system is what is important for logic and not the individual symbols.

And anyway, is it really possible that in logic I should have to deal with forms that I can invent? What I have to deal with must be that which makes it possible for me to invent them.

5.556 There cannot be a hierarchy of the forms of elementary propositions. We can foresee only what we ourselves construct.

5.5561 Empirical reality is limited by the totality of objects. The limit also makes itself manifest in the totality of elementary propositions.

Hierarchies are and must be independent of reality.

5.5562 If we know on purely logical grounds that there must be elementary propositions, then everyone who understands propositions in their unanalysed form must know it.

5.5563 In fact, all the propositions of our everyday language, just as they stand, are in perfect logical order.—That utterly simple thing, which we have to formulate here, is not a likeness of the truth, but the truth itself in its entirety.

(Our problems are not abstract, but perhaps the most concrete that there are.)

$$x,\ \Omega'x,\ \Omega'\Omega'x,\ \Omega'\Omega'\Omega'x,\ \ldots,$$

in the following way

$$\Omega^{0}'x,\ \Omega^{0+1}'x,\ \Omega^{0+1+1}'x,\ \Omega^{0+1+1+1}'x,\ \ldots.$$

Therefore, instead of '$[x, \xi, \Omega'\xi]$',
I write '$[\Omega^{0}'x, \Omega^{\nu}'x, \Omega^{\nu+1}'x]$'.
And I give the following definitions

$$0+1 = 1 \text{ Def.},$$
$$0+1+1 = 2 \text{ Def.},$$
$$0+1+1+1 = 3 \text{ Def.},$$
(and so on).

6.021 A number is the exponent of an operation.

6.022 The concept of number is simply what is common to all numbers, the general form of a number.

The concept of number is the variable number.

And the concept of numerical equality is the general form of all particular cases of numerical equality.

6.03 The general form of an integer is $[0, \xi, \xi+1]$.

6.031 The theory of classes is completely superfluous in mathematics.

This is connected with the fact that the generality required in mathematics is not *accidental* generality.

Logic's propositions are tautologies and say nothing. Any theory giving them content is false.

6.1 The propositions of logic are tautologies.

6.11 Therefore the propositions of logic say nothing. (They are the analytic propositions.)

6.111 All theories that make a proposition of logic appear to have content are false. One might think, for example, that the words 'true' and 'false' signified two properties among other properties, and then it would seem to be a remarkable fact that every proposition possessed one of these properties. On this theory it seems to be anything

but obvious, just as, for instance, the proposition, 'All roses are either yellow or red', would not sound obvious even if it were true. Indeed, the logical proposition acquires all the characteristics of a proposition of natural science and this is the sure sign that it has been construed wrongly.

The mistake: 'true' and 'false' as two properties among others, like yellow and red in roses.

6.112 The correct explanation of the propositions of logic must assign to them a unique status among all propositions.

6.113 It is the peculiar mark of logical propositions that one can recognize that they are true from the symbol alone, and this fact contains in itself the whole philosophy of logic. And so too it is a very important fact that the truth or falsity of non-logical propositions *cannot* be recognized from the propositions alone.

The mark of a logical proposition: its truth can be recognised from the symbol alone.

6.12 The fact that the propositions of logic are tautologies *shows* the formal—logical—properties of language and the world.

That they're tautologies shows the formal properties of language and the world.

The fact that a tautology is yielded by *this particular way* of connecting its constituents characterizes the logic of its constituents.

If propositions are to yield a tautology when they are connected in a certain way, they must have certain structural properties. So their yielding a tautology when combined *in this way* shows that they possess these structural properties.

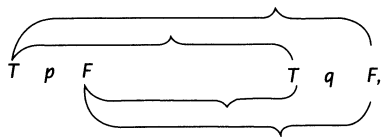
6.1201 For example, the fact that the propositions 'p' and '$\sim p$' in the combination '$\sim(p.\sim p)$' yield a tautology shows that they contradict one another. The fact that the propositions '$p \supset q$', 'p', and 'q', combined with one another in the form '$(p \supset q).(p):\supset:(q)$', yield a tautology shows that q follows from p and $p \supset q$. The fact that '$(x).fx:\supset fa$' is a tautology shows that fa follows from $(x).fx$. Etc. etc.

Wittgenstein's cases: p and not-p, then modus ponens. The tautology shows that q follows.

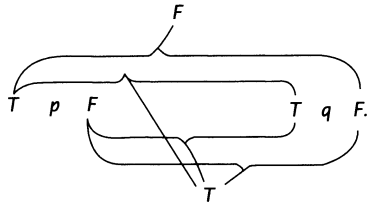
6.1202 It is clear that one could achieve the same purpose by using contradictions instead of tautologies.

A method for recognising tautologies, but only where no generality-sign occurs.

6.1203 In order to recognize an expression as a tautology, in cases where no generality-sign occurs in it, one can employ the following intuitive method: instead of 'p', 'q', 'r', etc. I write 'TpF', 'TqF', 'TrF', etc. Truth-combinations I express by means of brackets, e.g.



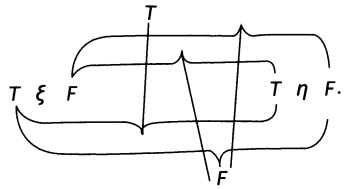
and I use lines to express the correlation of the truth or falsity of the whole proposition with the truth-combinations of its truth-arguments, in the following way



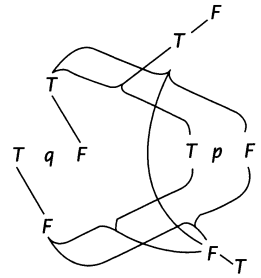
So this sign, for instance, would represent the proposition $p \supset q$. Now, by way of example, I wish to examine the proposition ~(p.~p) (the law of contradiction) in order to determine whether it is a tautology. In our notation the form '~ξ' is written as



and the form '$\xi . \eta$' as



Hence the proposition ~(p.~q) reads as follows



If we here substitute 'p' for 'q' and examine how the outermost T and F are connected with the innermost ones, the result will be that the truth of the whole proposition is correlated with *all* the truth-combinations of its argument, and its falsity with none of the truth-combinations.

6.121 The propositions of logic demonstrate the logical properties of propositions by combining them so as to form propositions that say nothing.

This method could also be called a zero-method. In a logical proposition, propositions are brought into equilibrium with one another, and the state of equilibrium then indicates what the logical constitution of these propositions must be.

6.122 It follows from this that we can actually do without

The further step: in a suitable notation, formal properties can be seen by mere inspection.

logical propositions; for in a suitable notation we can in fact recognize the formal properties of propositions by mere inspection of the propositions themselves.

6.1221 If, for example, two propositions 'p' and 'q' in the combination 'p ⊃ q' yield a tautology, then it is clear that q follows from p.

For example, we see from the two propositions themselves that 'q' follows from 'p ⊃ q.p', but it is also possible to show it in this way: we combine them to form 'p ⊃ q.p:⊃:q', and then show that this is a tautology.

6.1222 This throws some light on the question why logical propositions cannot be confirmed by experience any more than they can be refuted by it. Not only must a proposition of logic be irrefutable by any possible experience, but it must also be unconfirmable by any possible experience.

6.1223 Now it becomes clear why people have often felt as if it were for us to '*postulate*' the 'truths of logic'. The reason is that we can postulate them in so far as we can postulate an adequate notation.

6.1224 It also becomes clear now why logic was called the theory of forms and of inference.

6.123 Clearly the laws of logic cannot in their turn be subject to laws of logic.

(There is not, as Russell thought, a special law of contradiction for each 'type'; one law is enough, since it is not applied to itself.)

General validity isn't the mark of logic. An ungeneralized proposition can be a tautology too.

6.1231 The mark of a logical proposition is *not* general validity.

To be general means no more than to be accidentally valid for all things. An ungeneralized proposition can be tautological just as well as a generalized one.

The axiom of reducibility: if true, a fortunate accident. A world where it fails is imaginable.

6.1232 The general validity of logic might be called essential, in contrast with the accidental general validity of such propositions as 'All men are mortal'. Propositions like Russell's 'axiom of reducibility' are not logical propositions, and this explains our feeling that, even if they were true, their truth could only be the result of a fortunate accident.

6.1233 It is possible to imagine a world in which the axiom of reducibility is not valid. It is clear, however, that logic has nothing to do with the question whether our world really is like that or not.

Logic's tie to the world: names must have meaning, elementary propositions sense.

6.124 The propositions of logic describe the scaffolding of the world, or rather they represent it. They have no 'subject-matter'. They presuppose that names have meaning and elementary propositions sense; and that is their connexion with the world. It is clear that something about the world must be indicated by the fact that certain combinations of symbols—whose essence involves the possession of a determinate character—are tautologies.

That certain combinations of symbols are tautologies indicates something about the world.

This contains the decisive point. We have said that some things are arbitrary in the symbols that we use and that some things are not. In logic it is only the latter that express: but that means that logic is not a field in which *we* express what we wish with the help of signs, but rather one in which the nature of the absolutely necessary signs speaks for itself. If we know the logical syntax of any sign-language, then we have already been given all the propositions of logic.

All true logical propositions can be described in advance. So there can't be surprises in logic.

6.125 It is possible—indeed possible even according to the old conception of logic—to give in advance a description of all 'true' logical propositions.

6.1251 Hence there can *never* be surprises in logic.

6.126 One can calculate whether a proposition belongs to

logic, by calculating the logical properties of the *symbol*.

And this is what we do when we 'prove' a logical proposition. For, without bothering about sense or meaning, we construct the logical proposition out of others using only *rules that deal with signs*.

The proof of logical propositions consists in the following process: we produce them out of other logical propositions by successively applying certain operations that always generate further tautologies out of the initial ones. (And in fact only tautologies *follow* from a tautology.)

Of course this way of showing that the propositions of logic are tautologies is not at all essential to logic, if only because the propositions from which the proof starts must show without any proof that they are tautologies.

6.1261 In logic process and result are equivalent. (Hence the absence of surprise.)

6.1262 Proof in logic is merely a mechanical expedient to facilitate the recognition of tautologies in complicated cases.

6.1263 Indeed, it would be altogether too remarkable if a proposition that had sense could be proved *logically* from others, and *so too* could a logical proposition. It is clear from the start that a logical proof of a proposition that has sense and a proof in logic must be two entirely different things.

6.1264 A proposition that has sense states something, which is shown by its proof to be so. In logic every proposition is the form of a proof.

Every proposition of logic is a modus ponens represented in signs. (And one cannot express the modus ponens by means of a proposition.)