Metaphysics // Fall 2016

Handout 10

Mathematical applicability: Steiner, Colyvan

The notion of mathematical applicability. Steiner begins by reviewing previous attempts in the history of philosophy to understand mathematical applicability. The initial puzzle seems to be that mathematics examines the properties of non-empirical objects, yet its results prove remarkably useful in examining the properties of empirical objects and predicting their behaviour. Two major strategies emerge. One tries to resolve the puzzle. Thus, e.g., Kant claims that our empirical inquiry is necessarily informed by mathematical concepts. Another strategy is to dismiss the puzzle. Thus Berkeley argues that there is no mathematical applicability, since mathematics is incoherent to begin with. The only reason it is empirically useful is that its erroneous claims cancel each other out.

Steiner concludes by asking two question: (*a*) What do we do when we apply mathematics? (*b*) What is the meaning of the claim that mathematics is applicable in an empirical discipline?

Canonical empirical applications. Sometimes a mathematical theory was developed precisely with a certain empirical regularity. This is an instance of canonical empirical application of mathematics. One such application is the application of the mathematical theory of addition to counting the size of a collection of empirical objects.

Mereological sums. Consider the set $S = \{x_1, x_2, \dots, x_n\}$ and consider the mereological sum of the elements of this set: $S' = \langle x_1, x_2, \dots, x_n \rangle$. Suppose further that we think of physical bodies as maximal polyhedra. Then these bodies can play a role of a set element: they have parts, but those parts are not bodies themselves (those parts will not be maximal polyhedra). On the other hand, they can be collected into a mereological sum.

Thus it seems that mereological sums are an application of mathematical concept of set. But notice here that mereological sums would not help us if bodies constantly appeared and disappeared, or if they merged with other bodies, or if they were splitting up into parts too frequently.

Counting. How do we count the members of a given finite collection? Just by pointing to the objects and reciting numerals. The order of counting does not matters, as long as the collection is finite. Thus the ordinal numbers of that collection would be the same as the cardinal number.

Remark 1. Steiner says that counting an infinite collection would depend on the order of counting. This is a result in set theory, but notice that this has no relevance to our problem, since we cannot count, empirically, all members of an infinite collection.

Addition. This is an operation on cardinals, such that:

$$|X| = m,\ |Y| = n,\ X \cap Y = \varnothing,\ |X \cup Y| = m + n. \tag{10-1}$$

Suppose now that X' and Y' are mereological sums of the elements of X and Y. Then the mereological sum Σ is an 'application' of the union operation $\cup$, so that $\Sigma X'Y'$ is a 'model' or 'image' of $X \cup Y$. For this reason the cardinal sum $m + n$ delivers accurate predictions of counting the members of $X \cup Y$.

Conditions of application. We have talked of applying the mathematical concept of *addition* to the empirical question about the *size of the collection of physical bodies*. We see that this application depends on some facts. One is that the collection of bodies remains stable through time: bodies do not pop in and out of existence, do not fuse with one another, do not split. It is unclear, I think, how stable the collection should be.

Another condition is that the collection is finite. Only then ordinals and cardinals are the same. Now what if it is infinite? We can calculate the cardinality of an infinite set, but would that correspond to the number of objects in the collection? From Steiner's discussion the answer is not clear at all. Counting in different orders would deliver different cardinals. So we have not provided a rule for applying addition to that case. But one might argue that our question is far-fetched in the first place, because we do not know what it *is* to count an infinite collection.

Non-canonical empirical applications. Here we deal with the cases where mathematical concepts were prior and independently of their applications. Steiner gives a number of examples. Perhaps the most useful one would be an application of theory of conic sections (roughly, ellipses) to the description of planetary orbits. Kepler was able to utilise the already available mathematical theory to describe a physical phenomenon.

Where exactly is the problem here if there is any? It seems that the problem is *epistemic*: we want to understand how one practice with its internal epistemic rules R, with it being 'about' a set S of objects, can be *used* effectively within another practice with different epistemic rules R' and a different set S' of objects.

The simplest way to put this worry is to ask the question why chess is not applied to empirical science (so far? or never?).

Observe that this way of framing the problem presupposes a non-empirical mathematical ontology. But we can avoid this assumption, I think, and simply put the question in terms of epistemic rules alone.

Canonical non-empirical applications. Here we examine the question how a mathematical theory can be used to extract information about another theory, itself mathematical.

One example is the use of set-theoretic methods in understanding multiplication. However, it is the more advanced example, that of group theory, that is of greater interest. Group theory was originally invented by Galois to study the properties of algebraic equations. The first step in the physical interpretation of groups was made when groups were found to describe conservation laws in physics. Later on electron (or rather, its spin) was described by the group SU(2). This by itself is not very remarkable, so far as this merely delivers a convenient description that is dispensable.

But then the mathematical properties of SU(2) were utilised to speculate about the existence of other particles (corresponding to the three-dimensional representation of that group). These speculations have been confirmed—even though, for example, that there is still no known *physical* connection between the spin of electron and the spin of nucleon, one of the particles so described (i.e. the spin of proton and neutron).

Another strange thing happened when it was speculated that there are particles describable by the group SU(3). These speculations similarly were proposed in the first place as reflections on group theory. Merely by exploring the properties of a group physicists were able to speculate about physical existence.

Steiner calls this procedure 'canonical' and 'non-empirical' application. It is not clear to me why it is canonical, since group theory was originally invented for the study of equation, rather than symmetries (which makes it relevant in quantum physics). It is also not clear why it is non-empirical. Ostensibly the reason is that description of empirical phenomena is 'induced' by the description of mathematical structures (647). But the end result is still empirical. The whole point of this application is not to make a mathematical discovery (e.g., SU(3) and its representations can be studied separately from any use in quantum physics).

The spectre of Pythagoreanism. Pythagoras was known for his belief that all nature is mathematics. These results ostensibly confirm this view: by exploring mathematical theories you can discover facts about nature. The basic example of electron is instructive. It is not merely 'a very small bit of matter'. In the context of physical theory it is nothing but a representation of group-theoretic construction. Moreover, sometimes it is the *formalism* itself of the theories that allows one to extract predictions in physical science.

Logical applications. Consider the following inference:

(1) There are seven apples (here).
(2) There are five apples (there).
(3) $7 + 5 = 12$.
(4) Therefore, there are twelve apples (here and there).

A mathematical claim is used in logically deriving an empirical claims from other empirical claims. But what does mathematics, its objects, have to do with apples?

There is a logical and metaphysical sides to this question. The metaphysical aspect is that mathematical objects are too 'remote', that they should 'have nothing to do' with apples. The logical aspect is that there is an confusion here between the use of numerals as adjectives and nouns.

Both questions were addressed by Frege. His solution is based on the following equivalence:

$$\text{There are } n\ F\text{s if and only if } \#F = n, \qquad (10\text{-}2)$$

where F stands for concepts. Thus numbers would be associated with classes of concepts (rather than objects).

Example 2. The number of the concept APOSTLE is the extension (second-order concept) containing all those first-order concepts that are equinumerous with this concept. It will contain, for instance, the concepts TRIBE OF ISRAEL, KANT'S CATEGORIES, and $\lambda x.0 < x < 13$.

Remark 3. An alternative definition of numbers proposed by Frege is *implicit*:

$$F\text{s and } G\text{s are equinumerous if and only if } \#F = \#G. \qquad (10\text{-}3)$$