Metaphysics // Fall 2018

Handout 11

Truth, epistemology, metaphysics: Benacerraf

What is at stake. Right at the beginning Benacerraf announces the goals of his argument. Accounts of 661
mathematical truth were expected to meet two desiderata. They had to be able to identify the notion of truth that would be continuous with semantic notion of truth available for other discourses and would not be open to epistemological challenges. But these desiderata cannot be met simultaneously. If they can be met at all, one should be met at the expense of the other.

That is, some accounts can construct truth theories for mathematical statements, but only by ignoring the 662
question how such statements can be known. Other accounts, motivated exactly by epistemological concerns, can reinterpret mathematical statements in a way consistent with empiricist epistemology, but only at the cost of having 'truth theories' that do not give a proper account of truth (as we understand it from the semantic theory of truth).

Platonism and anti-platonism. Should mathematical statements be semantically interpreted in the same way as empirical statements? Benacerraf illustrates this question with the example in page 663. The statement:

There are at least three large cities older cities older than New York (11-1)

if taken at face value, involves quantification over cities, as well as the assignment of reference to the name 'New York'. What about the statement:

There are at least three perfect numbers greater than 17? (11-2)

Should we say that this statement, for it to be true, should also involve quantification over numbers and 664
assignment of reference to '17'?

Several influential views in the philosophy of mathematics, driven by epistemological considerations, resisted this move. Under the heading 'combinatorial view' Benacerraf explores Hilbert's later views. There are two levels in mathematics, the intuitive and the ideal. Statements of intuitive mathematics are verifiable directly by (finite) calculation. Typically they include proper names for numbers and can be formulated without the aid of quantifiers. Statements of ideal mathematics involve quantification over infinite domains. They are accepted not because there are particular verification procedures for establishing their truth, but rather wholesale—because of their pragmatically good contributions to the edifice of mathematics.

Example 1. The statement $1 + 1 = 2$ is a statement of intuitive mathematics. The statement 'There are infinitely many natural numbers' is a statement of infinite mathematics.

Remark 2. Hilbert's views are set out in the essay 'On the infinite'.

However the theory of truth for infinite mathematics is construed, it is clear that semantics for its quantifiers will be different from the semantics for quantifiers of the ordinary empirical language. Benacerraf says that giving such semantics would be 'very hard'.

On 'other' accounts, truth conditions of arithmetical statements are aligned with provability. A statement 665
provable in a formal axiomatic system is declared to be true. Hilbert himself espoused such an account early on in his career (arguably he defended it later on too, but the story gets complicated with the introduction of 'ideal elements'). Its other proponents include Carnap who defended it under the heading of 'conventionalism'. Indeed, this is what we saw earlier in the Carnap/Quine debate. There is no truth *simpliciter*, only truth in a given system. Having laid down the axiom, we proceed to manipulate them 'combinatorially' to generate theorems. Such manipulations impose truth or falsehood on the given statement, without a concern for satisfaction of its individual parts. Unlike the semantic notion of truth, this *syntactic* notion provides top-down explanations, whereby reference is explained by the prior assignment of truth value.

Remark 3. Platonists are obviously realists about mathematical objects. But some anti-Platonists might also insist that they are realists, but argue further that mathematical objects are not given to us the way material objects are. Both Platonist and anti-Platonists, of the sort considered by Benacerraf, are realists about the truth values of mathematical statements.

Remark 4. For Hilbert's early views, see especially his correspondence with Frege.

Two constraints. If there is a disagreement between platonists and anti-platonists, what constraints should
we impose on the preferred view? Benacerraf names two. First, the preferred view should show that its account 666
of truth corresponds to what we can recognise as 'truth'. In particular, we cannot simply *declare* that truth and provability (theoremhood) of mathematical statements are hereby synonymous. There must be a substantive explanation why these two notions extensionally equivalent, if indeed they are.

This constraint, Benacerraf observes, may be restated by demanding that the notion of mathematical truth is continuous with the 'general' notion of truth—a recognised notion in use in other disciplines and discourses. Since the main available notion of truth is that of Tarski's, this means that a theory of mathematical truth should yield a semantic notion.

The second constraint is that the preferred view must yield a plausible account of mathematical knowledge. 667
Once again, this means that the idea of mathematical knowledge must be continuous with the idea of knowledge in other disciplines and discourses.

We can see that the platonist accounts are driven by the concern to meet the first constraint, while anti- 668
platonist accounts are driven by the concern for the second constraint.

Costs and benefits of platonism. The main advantage of platonism is the availability of truth theories. 670
Platonism does not suggest any novel account of truth, and so can make use of the extant semantic theories, such as Tarski's. This in turn provides the platonist with all the necessary tools to account for logical inferences. As there is no contrast in this regard between mathematics and empirical sciences, the familiar tools of first-order logic stand ready for the use by the platonist.

The main cost of platonism: it cannot be integrated into a plausible account of knowledge. Supposing 670
those mathematical objects exist in the same way as empirical objects, how do we know anything about them?
With empirical objects, again, there is a causal account: our knowledge is justified by the fact of our causal 671f
interaction with these objects. Nothing of the sort can be assumed for mathematical objects.

This, then, is the 'integration challenge' (Peacocke's term): platonist metaphysics, for all its perceived naturalness primarily due to conservatism with respect to truth and inference, cannot be integrated into an acceptable epistemological account.

Comment on Gödel. The integration challenge is further illustrated with the quotation from the arch-platonist 674
Gödel. The talk of perceiving mathematical objects, of the axioms forcing themselves on the enquirer, rings empty, unless it is supplemented by an account of the *link* between the objects and the epistemic faculties of the subject. Whilst in empirical science there is a plausible candidate for explaining the link (the causal theory of knowledge), in mathematics one does not know where to start.

Remark 5. Penelope Maddy in her work on mathematical perception sought to bridge the gap. See also earlier work by Charles Parsons on mathematical intuition.

Problems with anti-platonism. The version of anti-platonism presented by Benacerraf is based on the 675
idea that mathematical knowledge is gathered from proofs. No mysterious mathematical perception is thus required. This idea is then extended: mathematical truth is also grounded in proofs. One sub-version is Hilbert's formalism (see Remark 4). The other one is conventionalism (philosophically, it is closely related to formalism): mathematical statements are true or false 'in virtue of' conventions. Axioms of the theory are supposed to be such explicit conventions.

One argument against conventionalism is Quine's. Since, in logic and mathematics, we allow infinitely 676
many statements to be true or false, we cannot characterise their truth values one by one. This would require infinitely many conventions. So truth, if conventional, must be generated by a general principle. Yet any individual assignment of truth following that principle would require inferential rules. But then these inferential rules must have validity *prior* to the specification of truth by our general principle—which is impossible.

All the same, according to Quine, if we have to take care only of a finite number of truths, conventionalism
is acceptable. Benacerraf protests: this notion of truth will be devoid of the characteristic referential clauses 677
built into Tarski's conception. The concepts of naming, predication, satisfaction, and quantification would be foreign to this anti-platonist conception of truth. It seems that the claim is: either the concept of truth is determined semantically, through objective relations between sentential ingredients and its referents (objects or properties). Or else the concept of truth 'sprinkled' on sentences, a mere monosyllable, is not a genuine concept of *truth* at all.

Intriguingly, Benacerraf also addresses Quine's claim of sentential priority. Recall that in Quine sentences 678
are judged true or false, and *on that basis* we determined reference of sentential parts, i.e. the ontological commitments of a theory (as well as its ideology). Benacerraf does not object to this claim in principle. Nevertheless he insists that for it to be defesible, one has to have a grip on the semantic concept of truth in the first place.

Remark 6. Quine's claim is an incarnation of Frege's context principle espoused in the *Foundations of Arithmetic*.