Metaphysics // Fall 2018

Handout 12

Mathematical existence: Putnam

THE PROBLEM STATED. Earlier we got a good synoptic view of the debate in Benacerraf's article. Putnam and Yablo in our readings focus specifically on the prospects for mathematical nominalism (of some sort).

Putnam begins by locating the debate in the context of explaining logical validity. Nominalists purport to disbelieve in mathematical entities (for the reasons charted by Benacerraf) and so propose to abolish reference to them. But now, how can they provide the notion of validity? To formulate this notion in (A) we use objectual quantification over classes. We can re-write by using substitutional quantification, whereby we quantify over words and sentences, as in (B). 9

Remark 1. The inference in (A) is the so-called 'Barbara' syllogism, the simplest logical inference in the Aristotelian taxonomy of inferences. Putnam's reasoning carries over even more straightforwardly to the case of a more familiar inference rule such as *modus ponens*.

Question 2. Give an example of a Barbara syllogism.

But even with (B), we have to make claims such as:

> For all *possible* substitutions of S, M, and P, if all S are M, and all M are P, then all S are P. (12-1)

So the nominalist is forced to quantify over possibilities which is no more palatable than quantification over mathematical entities. The nominalist may try to evade the difficulty by restricting substitution to a particular language. Instead of (12-1) we could write: 10 12

> For all substitutions of S, M, and P in a language L, if all S are M, and all M are P, then all S are P. (12-2)

The language L should be rich enough to have names for collections of things to be substituted for the variables S, M, and P. Yet no language can be so rich as to contain names for *all* collections of things, if there are infinitely many things (see Russell's paradox).

So then, instead of one formalised language L, the nominalist may propose a notion of validity in different formalised languages. This brings us back to the earlier problem of dealing with all *possible* languages. 13

TRUTH. Can there be a nominalistic notion of truth? Initially Putnam raises this problem: inscriptions and sentence-tokens are not appropriate candidates for being truth-bearers. It is rather what they express, the propositions, that are. Yet propositions are suspect ontologically no less than mathematical entities. The nominalist may reply that truth is not a unary predicate, but a tertiary relation between inscriptions, epistemic subjects, and temporal instants (as in Putnam's (7)). 13 18

But this response is spurious. Putnam complains that there must be some explanation of this relation, since it is not a primitive one. Now I think there are more devastating objections. First, falsehood can also be rendered in (7). On its own (7) does not show any difference between true and false sentences (or statements?). Second, the truth of sentences or statements appears to change with time. This was indeed one of Scholastic doctrines, but too incredible to even attempt to defend. 21

THE FAILURE OF NOMINALISM IN SCIENCE. Nominalists have the ambition to eliminate reference to abstract entities from the discourse of mathematics. Putnam aims to show that nominalism is inadequate for empirical science as well.

Consider Newton's law of gravitation which we will assume to describe an objective law governing the behaviour of material bodies;

$$F = \frac{Gm_a m_b}{d^2}. \tag{12-3}$$

On the face of it, this statement is incompatible with nominalism, so far as it commits us to the existence of numbers. This commitment to mathematical entities is the first obstacle. But there is trouble with representing physical entities in a way acceptable to nominalism, such as forces, masses, and distances.

Yet couldn't a nominalist come up with his paraphrase in principle? There has been as yet no translation of a 'measurement statement', such as 'the force F is q'. Putnam then gives an argument to the effect that any nominalistic paraphrase requires a universe of infinitely many physical objects.

PUTNAM'S ARGUMENT SIMPLIFIED. The law (12-3) correlates arbitrary magnitudes. So we must be able to say:

For every distance d, masses m_1 and m_2, the force is f. (12-4)

The nominalized paraphrase is characterized by the fact that physical objects play the role of mathematical entities. If there are finitely many individuals, then, in the nominalized language, there must be only finitely many ways of, for example, saying:

The distance between a and b is n metres. (12-5)

Which means that the paraphrase will fail, unless there is an assumption of infinitely many objects.

INDISPENSABILITY AND QUEER QUESTIONS. There is nothing deviant about philosophical claims of existence. The alleged deviance admits only of a *circular* argument.

Secondly, while there may be 'deviant' pure philosophical claims of existence, mixed claims are accepted by all as non-deviant. That is:

$$\exists x(x \text{ is a number})$$

is supposedly deviant, but

$$\exists x(x \text{ is a number} \,\&\, x \text{ is prime})$$

is not deviant. But then ordinary language is not *deductively closed*. Some statements belong to it, but their logical consequences do not. To close it, we have to admit into it exactly the alleged deviant sentences.

THE FAILURE OF FICTIONALISM. Putnam notes one bad reason for the rejection of fictionalism: the dogma of verificationism. It is not logically impossible for us to be brains in a vat, or be directed by the malicious demon. The right reason, to cut the story short, is that the reasons directing us to accept a theory *qua* a theory are the very same reasons directing us to accept the existence of mathematical entities.

YSB