# Metaphysics // Fall 2019

## Handout 4

## Truth by convention III: Quine

**Mathematics again.** We now return to the subject of reducing mathematics to logic. Quine usefully summarises different ways discussed earlier of reducing geometry to logic via Huntington's 267:2 postulates. Conditional reduction, whereby every geometrical truth was seen merely as a theorem in axiomatic system, failed, because it was nothing more than a simple 'renaming'. Literal reduction, in which geometrical terms were defined contextually, has failed, because it couldn't do justice to the antecedent traditional use of geometrical statements (the baseball example). The only acceptable reduction was the one in *Principia* which directly identifies geometrical expressions with set-theoretic ones.

**Conventional reduction.** But now we may try another way, the 'conventional' way. Suppose 267:1 we have the conventions (I)–(III) to which we add further conventions for predicate calculus. We have the conventions (I)–(VII) in total. We now lay down the convention (4-1a) that all Huntington postulates are true. Because we have already said that all geometrical truths can be derived from the postulates, we also have (4-1b).

(4-1) a. Let 'Hunt(sphere, includes)' be true.

b. 'If Hunt(sphere, includes) then Φ(sphere, includes).'

Geometry is not part of logic. But, just like logic, it is true by convention. What about this method? 268:1,2

In the first place, the same may be done for empirical disciplines. There is nothing specifically 268:3 'mathematical' or 'non-empirical' in our additional conventions. Still, if we do manage to define some 268:4 empirical expression in this conventional manner, then there will be 'no question' that it belongs to mathematics. That is, disciplines so far considered empirical would now be considered mathematical, 'formalised'.

But then there are of course many notions in empirical disciplines that are conventionally defined 268:4 since the days of Galileo. Some of these definitions occur in science textbooks ('momentum', 'electron', 'cell', 'gene') and are at the heart of normal presentations of scientific theories, while other are introduced by philosophers and philosophically-minded scientists.

Quine envisages a thorough reduction of all terms in the empirical discipline and their specification 269 through a chain of conventions. In this way all sentences containing these terms will be true by convention. In the process there there may be a question of conformity to traditional use, the difficulty 269:2 that dogged similar reductions earlier on. Terms like 'event', 'time', or even 'momentum' are used in the vernacular. And what guarantee is there that the conventions will respect that use? Still, we may assume that there is some agreement on their conformity.

Such 'conventional' truths, however, will be nothing like what we could expect from logical or 269:2 mathematical truths. They could very well be rejected, because empirical observations show their convenience. Quine does not tell us exactly how these conventions are rejected by observation. This is a very complex and delicate point, and I suspect that Quine at the time (1936) was not aware just how complex and delicate it was. It is easy to see why internal considerations of convenience may force a change of conventions. For example, Newton thought of force intuitively as a 'power of resisting' (Definition III of the *Principia*). Euler in his later presentation of mechanics adopted the Second Law as a definition of force, dispensing with such unclear intuitive ideas going all the way back to Aristotle. But how can an *empirical* observation force a similar change?

I suspect that Quine was thinking along these lines. Let 'mass' in a theory $T$ be understood as a 'quantity of matter', a permanent characteristic of the body, in particular independent of that body's velocity. Now certain observations may clash with certain predictions delivered by $T$. We then replace $T$ with $T'$. But in order to accommodate these observations in $T'$ it is found convenient to treat 'mass' as a characteristic that *does* depend on velocity. This leads to a different convention of the use of the term 'mass'. There are many unclear elements in this picture, but I believe that some such picture was on Quine's mind at the time.

*Remark* 1. Alternatively, we may define 'mass' operationally in Mach's fashion. But all sorts of empirical considerations, such as observations over systems of four bodies, may force us again to abandon the definition. (Ignore this remark if it makes no sense to you.)

The attempt to see mathematics and logic as true by convention there are three possibilities, Quine 270:1
says, all of them unappealing. Either they are true by convention in the same way as empirical disciplines (the claim is empty). Or the theorist decides to discriminate arbitrarily against those other disciplines (the claim is uninteresting). Or it is a general practice to adopt explicit conventions for these two disciplines (the claim is false).

**An important digression.** [Quine pauses to inform us about the real distinction between neces- 270:2
sity and contingency, apriority and aposteriority, logic/mathematics and empirical disciplines. His view here anticipates the later views in the *Two dogmas of empiricism* and in *Carnap and logical truth*, but is not yet that radical. To be filled in in class. . . .]

**The regress argument.** Quine returns to the main thread of the argument. He now formulates 270:3
the second objection against conventionalism: If conventions are laid down as general statements, then to infer anything from them requires logic in turn. How can we show this?

We have a convention (II) for *modus ponens*. We re-write it as (4-2a). Then we follow Quine's steps. The trick is to try to infer the truth of (4-2d) which is the consequent of statement (4-2c) which is itself true by Convention (I). We need a further convention (4-2f) which belongs to predicate calculus. It rids us of the universal quantifier in the Convention (II′), so that we could conduct an inference for particular statements.

(4-2)
a. For all $x, y, z$: if $x, z$ are true and $z$ is the statement 'If $x$ then $y$', then $y$ is true. [Convention (II′)]
b. $p \to (\sim p \to q)$: If time is money, then if time is not money, then time is money. [(3)]
c. $(p \to q) \to ((q \to r) \to (p \to r))$: If if time is money then if time is not money then time is money then if if if time is not money then time is money then time is money then if time is money then time is money. [(5)]
d. $(q \to r) \to (p \to r)$: if if if time is not money then time is money then time is money then if time is money then time is money. [(6)]
e. (3) and (5) are true: (5) is 'If $p$ then $q$', where $p$ is (3) and $q$ is (6). [(8)]
f. $\forall x\phi \to \phi(t)$: If $x$ occurs in a sentence $\phi$, then it can be replaced there with a term $t$. [Convention (IV)]

The regress follows once we observe that, for every instance of a derivation we need another convention that would certify that derivation. It is helpful to simplify Quine's reasoning here somewhat. Here is one attempt (4-3).

(4-3)
a. $p$ and ⌜if $p$ then $q$⌝ stand in a certain relation $\mathfrak{R}$ to $q$. [intuitively $\mathfrak{R}$ is the relation of *modus ponens*]
b. For all $x, y, z$: if $x$ and $y$ stand in the relation $\mathfrak{R}$ to $z$, then $x$ and $y$ jointly imply $z$. [all instances of $\mathfrak{R}$ are 'implicative', or 'entailments']
c. If $p$ and ⌜if $p$ then $q$⌝ stand in the relation $\mathfrak{R}$ to $q$, then $p$ and ⌜if $p$ then $q$⌝ jointly imply $q$. [from (4-3b) and repeated applications of Convention (IV)]
d. $p$ and ⌜if $p$ then $q$⌝ jointly imply $q$. [from (4-3a) and (4-3c)]

The problem is that the last step itself relies on recognising the relation $\mathfrak{R}$. The lesson is that, no matter what conventions you adopt, as long as you *infer* anything from these conventions, your inference must itself be part of logic—that is, there will be a further convention to introduce. And when you infer—*infer*!—anything again by using that further convention, then you will have to introduce yet another convention. Hence regress.

As Quine notes, another way to interpret the problem is by looking at the meaning of primitive 271:4
terms. The conventionalist would have us believe that logical constants have no meaning prior to conventional assignments, and that only those assignment impute meaning to them. But the application of those conventions (that is, using them in inferences) is only possible once the meaning is recognised independently. If it is not, then we are in need of further conventions to specify

how initial conventions are to be applied. And to apply these further conventions, we will need further-further conventions, and so on.

This sort of regress will disappear if conventional truth assignments are made piecemeal, since 272:2
there would then be no need to specify the regress-prone logical rules. But, as we have seen, this strategy founders on the infinity of tasks needed to be completed (see Handout 3).

**Implicit conventions.** The final bit concerns the option of adopting conventions implicitly. So 272:3
far the conventionalist was presented as someone who lays down a convention explicitly, and this laying down gives the warrant to the use of logical constants, rules of inference, and eventually truth value assignments. But one might instead think of conventions by analogy with grammar. (Educated) native speakers use their language grammatically. This is what, indeed, grammatical use is—the use by native speakers. Grammar rules merely *codify* this use. On their own, they provide no warrant. Rather, they make it easier to explain why a given utterance (say) is grammatical by linking it perspicuously to the native use. The explicit rules provides no warrant—which rests instead with native use.

Quine voices sympathy with this idea. But he insists that this would not count as 'conventionalism'. 273:2
The main reason, I read Quine as saying, is that conventions were supposed to have 'explanatory force'. Once they are deprived of that force, what philosophical role do they to play? Moreover, (I believe) he claims, with implicit conventionalism the difference between one convention and another would only be a matter of more or less firm acceptance, of difference in linguistic dispositions. But then the old problem that has been popping up throughout the paper resurfaces again: how do we distinguish between the supposedly conventional (≈ a priori, analytic) logic and mathematics and the supposedly non-conventional (≈ a posteriori, non-analytic) empirical disciplines?

YSB