Philosophy of Science // Fall 2016

Handout 3

Conventionalism in geometry: Poincaré

Earlier views on the status of geometry. Geometry has always been conceived as a deductive discipline: theorems logically follow from axioms. But what is the epistemic status of axioms? We can distinguish three prominent answers given before the dawn of the relativity theory: rationalist (most popular), empiricist (Mill, Helmholtz(?), Mach), and Kantian (popular in Germany).

> I discover innumerable particulars respecting figures, numbers, motion, and the like, which are *so evidently true, and so in accord with my nature*, that when I now discover them I do not so much appear to learn anything new as to call to memory what was before in my mind, but to which I had not hitherto directed my attention. (Descartes, *Med.* V.4)

> The peculiar accuracy, supposed to be characteristic of the first principles of geometry, appears to be fictitious. . . . When it is affirmed that the conclusions of geometry are necessary truths, the necessity exists in reality only in this, that they correctly follow from the suppositions from which they are deduced. These suppositions are so far from being necessary that they are not even true; they purposely depart, more or less widely, from the truth. . . . It remains to inquire, what is the ground of our belief in axioms—what is the evidence on which they rest? I answer, *they are experimental truths; generalizations from observation*. The proposition, Two straight lines can not inclose a space—i.e. that two points determine just one straight line—is an induction from the evidence of our senses. (Mill, *System of Logic* II.5.1,4)

The two quotations set up nicely a metaphysical conundrum. Descartes the rationalist can examine 'with his mind' the properties of an imaginary triangle. What can he know, then, of the properties of a real physical triangle? Thus open the floodgates of scepticism. By contrast, Mill the empiricist begins with the observations of a physical triangle. OK, but how will geometry differ from any empirical discipline then? In the first place, it is absurd to admit no difference. Also, we are again deprived of geometrical certainty.

Kant aims to escape the conundrum. His slogan is: 'Space is a form of pure intuition.' Claims of geometry are established in our imagination. They require no recourse to experience: they are products of our own mind. On the other hand, there is no gap between, say, triangles in our imagination and triangles in reality. Objective reality is limited to objects conforming to the rules set up by our own mind.

Crucial consequences of the Kantian approach: (1) Euclidean geometry is the only one intuitable. (2) Therefore, it is *the* true geometry in our world. (3) And *necessarily* true as well. All these claims were challenged in the later debates.

Alternative geometries. By rejecting the Fifth Postulate, we can construct non-Euclidean geometries. It is no accident that Lobachevsky's results were met with ridicule (in Russia). Since empiricism was rarely if ever articulated, the traditional approach was that the Euclidean geometry with all of its axioms was necessarily true. Thus, to reject one of the axioms, is to reject something that is self-evidently true (necessity and certainty coming together). But there is a more precise way to put the worry: if you reject one of the axioms, you will eventually entangle yourself in contradictions. Thus, the first task is to show that non-Euclidean geometries are *consistent*.

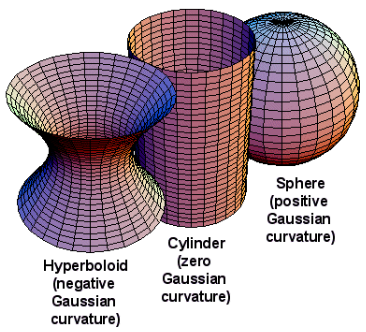

Figure 1: Curvatures

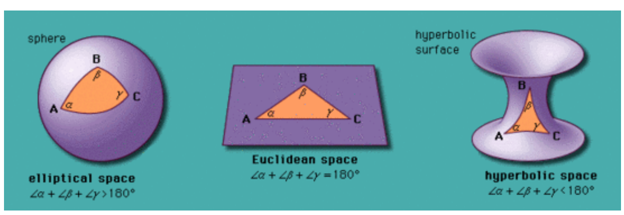

Figure 2: Sum of angles in a triangle

Conventionalism: translatability. While he mentions different approaches, Poincaré's preferred method for proving consistency is the method of relative consistency. Historically, the same method was adopted by Lobachevsky. Assuming that the Euclidean geometry is consistent, we can translate its statements into statements of non-Euclidean geometry. This is the main positive argument Poincaré provides for conventionalism (in our text).

Let us mention several objections to Poincaré's argument. (1) The label of 'conventionalism' does not sit well with the usual meaning of convention, which is something that is adopted not through a rational procedure. (2) Conventions can be thought as products of chance conditions (e.g., the choice of names). This is not the case of geometry. (3) The analogy with linguistic translation is not sufficiently developed. (4) And where is the argument for the conventionality of applied geometry? In fact the distinction is never quite fleshed out.

Negative arguments for conventionalism. The argument against empiricism is familiar: geometry is not approximate. The argument against apriorism is also familiar to us from Helmholtz: we can conceive non-Euclidean geometries. One example is the spherical Flatland, and the other is the heated sphere.

Question 1. Does Poincaré have an argument against the Kantian version of apriorism?

The heated sphere. Suppose the world is enclosed in a large sphere and is subject to certain (weird) laws. The temperature is not uniform: it is greatest at the centre, and gradually decreases towards the circumference of the sphere, where it is absolute zero. If R be the radius of the sphere, and r the distance of the point considered from the centre, then absolute temperature will be proportional to $R^2 - r^2$. Expansion (dilation) of a body is proportional to its absolute temperature. Finally, a body transported from one point to another of different temperature is *instantaneously* in thermal equilibrium with its new environment.

In this sphere, then, a moving object will be smaller and smaller as it approaches the circumference of the sphere. What consequences shall we derive about the geometry of the enclosed world? To its inhabitants the world will appear infinite. As they approach the surface of the sphere they become colder, and at the same time smaller and smaller. They can never reach the boundary of the sphere. With some additional assumptions, one can show that light trajectory will be circular (rather than rectilinear). And if they are never able to detect the global heating force at the heart of the sphere, their geometry will be non-Euclidean.

Conventionalism? What are we supposed to conclude from this thought experiment? An empiricist, like Mill, would conclude that the choice of geometrical axioms is an empirical matter. After all, that is how we describe the procedure through which the sphere inhabitants choose their geometry. Poincaré sees the matter differently. The sphere inhabitants indeed will choose a non-Euclidean geometry. But geometry is not an experimental science, since its objects are not physical objects. Its objects are ideal solid objects. No actual physical object is such a solid.

The contrast Poincaré intends to draw between geometry and natural science can only stand if we think of natural science as based on inductive generalisations. But we may think of it—say, along with Helmholtz—as similarly based on the desire to better arrange our representations. In this sense a physical theory would also be chosen for its 'convenience'. Furthermore, as we are going to see immediately, so far as a physical theory admits higher mathematical machinery, its objects can also be judged 'ideal'. A much more compelling elaboration of conventionalism, together with a crisper analysis of the heated sphere example, we shall find in Reichenbach.

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