Week 3

The nature of geometry

Poincaré, Einstein, Reichenbach

Slides for the lecture *Philosophy of Science* on 30 September 2014

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Outline

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1 The status of axioms: historical background

The appeal of geometry

- (Review our earlier discussion of Helmholtz.)
- Geometry has always been conceived as a deductive discipline: theorems logically follow from axioms.
- But what is the epistemic status of axioms?
- We can distinguish three prominent answers given before the dawn of the relativity theory: rationalist (most popular), empiricist (fairly marginal), and Kantian (popular in Germany).

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Rationalism and empiricism

> I discover innumerable particulars respecting figures, numbers, motion, and the like, which are *so evidently true, and so in accord with my nature*, that when I now discover them I do not so much appear to learn anything new as to call to memory what was before in my mind, but to which I had not hitherto directed my attention. (Descartes, *Med.* V.4)

> The peculiar accuracy, supposed to be characteristic of the first principles of geometry, appears to be fictitious. ... When it is affirmed that the conclusions of geometry are necessary truths, the necessity exists in reality only in this, that they correctly follow from the suppositions from which they are deduced. These suppositions are so far from being necessary that they are not even true; they purposely depart, more or less widely, from the truth. ... It remains to inquire, what is the ground of our belief in axioms—what is the evidence on which they rest? I answer, *they are experimental truths; generalizations from observation*. The proposition, Two straight lines can not inclose a space—i.e. that two points determine just one straight line—is an induction from the evidence of our senses. (Mill, *System of Logic* II.5.1,4)

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Difficulties of both

- The two quotations set up nicely a metaphysical conundrum.
- Descartes the rationalist can examine 'with his mind' the properties of an imaginary triangle.
- What can he know, then, of the properties of a real physical triangle?
- Thus open the floodgates of scepticism.
- By contrast, Mill the empiricist begins with the observations of a physical triangle.
- OK, but how will geometry differ from any empirical discipline then?
- In the first place, it is absurd to admit no difference.
- Also, we are again deprived of geometrical certainty.

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Kant on geometry

- Kant aims to get out of the conundrum.
- 'Space is a form of pure intuition': claims of geometry are established in our imagination.
- They require no recourse to experience: they are products of our own mind.
- On the other hand, there is no gap between, say, triangles in our imagination and triangles in reality.
- Objective reality is limited to objects conforming to the rules set by our own mind.

Crucial consequences
(1) Euclidean geometry is the only one intuitable. Therefore, (2) it is the true geometry in our world, and (3) necessarily so. All these claims were challenged in the later debates.

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2 Poincaré on non-Euclidean geometries

Alternative geometries

- By rejecting the Fifth Postulate, we can construct non-Euclidean geometries.
- It is no accident that Lobachevsky's results were met with ridicule (in Russia).
- The traditional approach was that the Euclidean geometry, all of its axioms, were necessarily true.
- Thus, to reject one of them, is to reject something that is self-evidently true (necessity and certainty coming together).
- But there is a more precise way to put the worry: if you reject one of the axioms, you will eventually entangle yourself in contradictions.
- Thus, the first task is to show that non-Euclidean geometries are *consistent*.

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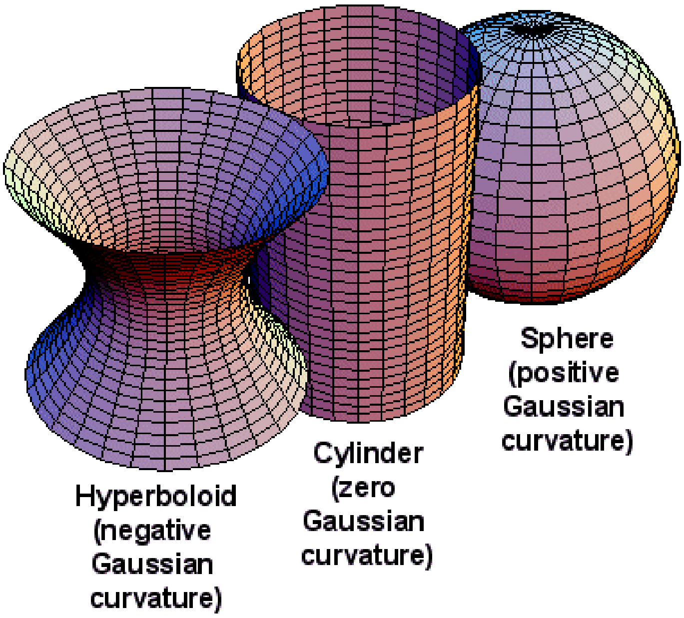
The three spaces: curvatures



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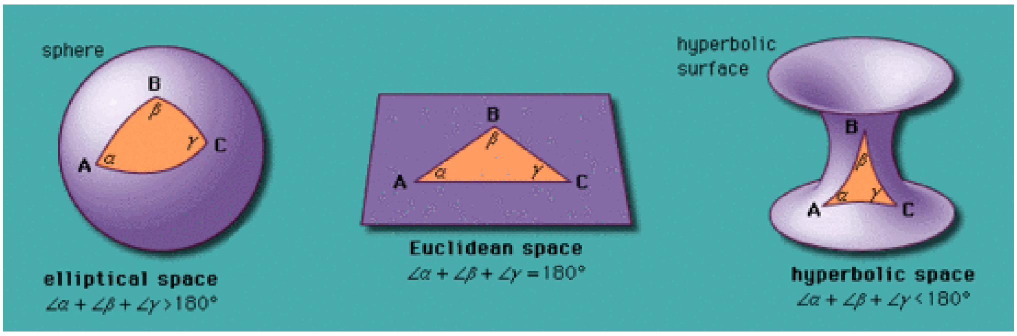
The three spaces: triangles



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3 Conventionalism

Translatability and conventionalism

- While he mentions different approaches, Poincaré's preferred method for proving consistency is the method of relative consistency.
- Historically, the same method was adopted by Lobachevsky.
- Assuming that the Euclidean geometry is consistent, we can translate its statements into statements of non-Euclidean geometry.
- This is the main positive argument Poincaré provides for conventionalism.

Objections

(1) This does not sit well with the usual meaning of convention, which is something that is adopted not through a rational procedure. Conventions can be thought as products of chance conditions. This is not the case of geometry.
(2) The analogy with language is not sufficiently developed.
(3) Where is the argument for conventionality of applied geometry? In fact the distinction is never quite fleshed out.

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Negative arguments for conventionalism

- The argument against empiricism is familiar: geometry is not approximate.
- The argument against apriorism is also familiar to us from Helmholtz: we can conceive non-Euclidean geometries.
- One example is the spherical Flatland, and the other is the heated sphere.

Question
Does Poincaré have an argument against the Kantian version of apriorism?

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Poincare's sphere

- Suppose a world is enclosed in a large sphere and is subject to certain (weird) laws.
- The temperature is not uniform: it is greatest at the centre, and gradually decreases towards the circumference of the sphere, where it is absolute zero.
- If R be the radius of the sphere, and r the distance of the point considered from the centre, then absolute temperature will be proportional to $R^2 - r^2$.
- Expansion (dilation) of a body is proportional to its absolute temperature.
- Finally, a body transported from one point to another of different temperature is *instantaneously* in thermal equilibrium with its new environment.

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Poincare's sphere II

- In this sphere, a moving object will be smaller and smaller as it approaches the circumference of the sphere.
- What consequences shall we derive about the geometry of the enclosed world?
- To its inhabitants the world will appear infinite.
- As they approach the surface of the sphere they become colder, and at the same time smaller and smaller.
- They can never reach the boundary of the sphere.
- With some additional assumptions, one can show that light trajectory will be circular (rather than rectilinear).
- And if they are never able to detect the global heating force at the heart of the sphere, their geometry will be non-Euclidean.

Question
Reflect on this example further.

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4 Einstein on geometry

Mathematical applicability

- Einstein begins with this question: how can mathematics be applied to reality?
- Depending on what your views of mathematics are, this can mean a familiar epistemological question: how does our thought represent reality?
- Or it can mean: how can statements about abstract mathematical objects be used in empirical predictions?
- Einstein's beautiful response: 'As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.'
- We think of axiomatic (pure) geometry as based on arbitrary axioms.
- To be able to apply geometry to the physical world, we should introduce a correlation between axiomatic geometry and practical (applied) geometry.

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The solids postulate

- Solid bodies of ordinary experience are those that undergo changes of position, but not changes of state (of quality).
- Hence the postulate: 'Solid bodies are related, with respect to their possible dispositions, as are bodies in Euclidean geometry of three dimensions.'
- These bodies that are so related Einstein calls 'practically rigid'.
- Are there such bodies in nature?
- This can only mean: which bodies in nature can play the role of practically rigid bodies?
- The assumption we make is that measuring rods act as practically rigid bodies.
- That is, measuring rods are unaltered under transport.

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Geometry as empirical science

- According to Einstein, Poincaré made a key mistake when he claimed epistemic priority for the Euclidean geometry.
- He could only have talked about the *pure* Euclidean geometry.
- But experience favours no pure geometry.
- With the solids postulate, however, we can come to the conclusion, as in GTR, that the practically rigid bodies do not obey the laws of the Euclidean geometry.

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5 Reichenbach on definitions

Conceptual and coordinative definitions

- Conceptual definitions aim to define concepts by *reducing* it to other concepts.
- This is the type of definitions familiar, for example, from the Euclidean geometry.
- Reichenbach claims there is another family of definitions practised in physics: coordinative definitions.
- They aim at *coordinating* concepts with physical objects.
- They cannot be seen as explications of meaning.
- (The terminology of 'coordination' can be replaced by 'correlation' or 'correspondence'.)
- Basic coordinative definitions are arbitrary (though that's not the case with all such definitions).

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Examples

Conceptual definitions
A *bachelor* is unmarried man.
A *line* is length without breadth. (Euclid, *Elements*, Book I, Def. ii)
A *triangle* is a figure formed by three right lines joined end to end. (*Elements*, Book I, Def. xx)

Coordinative definitions
A *unit* is a distance which, when transported along another distance, supplies the measure of this distance.
The length of *one meter* is the length of the rod in Paris.

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Relativity and rods

- Once a coordinative definition is in place, we have a (by now familiar) problem of measuring lengths at different locations.
- We must have assurances that no unduly deformations occur along the way.
- But our tests can only rule out local deformations: if only one rod is deformed, we will verify this by using *another* rod.
- This will not be verifiable in the case of global deformations whereby every object in the universe contracts or expands in exactly the same proportion.

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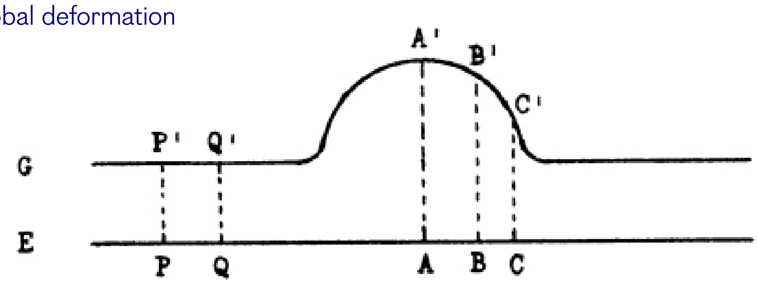
6 Universal force

Global deformation



- We have two surfaces G and E: one is with a hump, and the other is a (Euclidean) plane.
- Suppose a certain force acts on the surface E, so that every object there becomes in length equal to the shadow projected from the surface G.
- Suppose further that this force acts on different materials in the same way and that there is no protection from its influence.
- What kind of geometry would the E-people attribute to their surface? 20

Convention and cognition

- The thought experiment shows the need for conventions in addition to observations.
- It is an observable fact that our rods are equal in length locally (in the same place).
- When they are transported, it becomes an assumption that the length is preserved.
- That is, whether $AB = BC$ is not a matter of cognition and verification, but of coordinative definition.
- That is, two physical objects at two distinct locations are *defined* as having the same length.

Question
Reichenbach further says that the concept of equality of length is *not* thereby defined. Is this true? Why? 21

7 Falsification and pragmatism

Falsification

- Our choice of one single rod as a measuring standard depends on the physical behaviour of the rods.
- There could be a world where there the uniformity in behaviour were violated.
- Then we would have liked to avoid the definition of length in terms of one standard Parisian rod.
- (The modern metrical standard is different, but we let us keep using the Parisian standard for simplicity.)
- So we could have set up measuring rods in every location.
- And then, when the rods were compared locally, their lengths would not have matched.
- That, however, would not have refuted our original coordinative definition. 22

Criteria of choice

- It turns out, therefore, that our scientific theory relies, in its foundation, on coordinative definitions.
- They are left at our discretion.
- There is no sense in which a definition can be true or false.

- So how can we choose which definition to use?
- Our choice is pragmatic.
- A better definition is logically simpler.
- It also requires fewer adjustments in other fields of enquiry.
- A consequence here is that the scientific theory itself can no longer be declared true or false, if these notions are taken in the sense of 'corresponding to reality'. 23

8 Solidity

Shape

- Another matter to be resolved by coordinative definitions is *shape*.
- There is a good reason to distinguish changes through displacement and changes through deformation.
- The first ones can be corrected by changes in position, but the latter cannot—unless we manage to isolate smaller parts of a deformed body and attribute to them changes of displacement.
- In the transportation of measuring rods it is assumed that they do not change shape.
- Indeed, without this assumption there would be no geometry. 24

Solid rods

- Measuring rods, therefore, are taken as *solid*.
- What assurance can have of this fact?
- Again, we can verify their solidity if they are under the influence of differential forces.
- But there will be no assurance if they are influenced by universal forces. 25

9 Relativity of geometry

Technical and logical impossibility

- Reichenbach then addresses the problem of subjectivity.
- The objection, as usual, is that we cannot determine changes (in rod's shape or length) because of technical or cognitive limitations.
- But there is a way to determine them *objectively*.
- Answer: there is no way to say whether the Parisian rod is *really* one meter long.
- Secondly, if the problem were technical, approximations were possible. But there is no sense in which the geometry of the surface E above is approximately Euclidean, or approximately non-Euclidean.

Remark (for philosophers?)
Call the rod 'Harry'. Consider the statement 'Harry is one meter long.' Is it a priori? necessary? analytic? 26

Relativity of geometry

- Perhaps some people would maintain that the geometry of our world is Euclidean.
- Not that there are many such people left in academic circles now compared to 1928!
- Perhaps some Kantians might be desperate because of the special role assigned by Kant to the Euclidean geometry.
- The answer is that our measurements show that the geometry of our world could be Euclidean if we allow the presence of a certain *universal* force (that is, gravity).
- But alternatively, we can ignore the presence of that force and instead use non-Euclidean measurements.
- In any case, there is no ground for maintaining the a priori character of geometry. 27

Testability

- Perhaps it might further be objected that the Euclidean geometry is to be preferred because it is simpler.
- But this is a grave mistake.
- Formal simplicity of the mathematical theory 'Euclidean geometry' is no warrant for its acceptance as *physical* geometry.
- We apply pragmatic criteria to the combination $G + F$ of geometry and a physical theory of the universal force.
- The assumption $F \neq 0$ may result in fairly complex calculations.
- In general: what is testable is the set of actual measurements together with coordinative definitions (that would imply a choice of geometry).
- An important consequence is *holism*: results of measurements can be accepted or rejected only given the background theory going all the way to coordinative definitions.

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