THE LOGIC OF MODERN PHYSICS

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IN HARVARD UNIVERSITY

New York
THE MACMILLAN COMPANY
1958

IT IS WELL TO REVIEW MACH'S DISCUSSION OF MASS BEFORE STUDYING THE PRESENT TEXT

CHAPTER I

BROAD POINTS OF VIEW

Philosophical analysis of fundamental concepts could have prepared the ground for relativity theory

WHATEVER may be one's opinion as to our permanent acceptance of the analytical details of Einstein's restricted and general theories of relativity, there can be no doubt that through these theories physics is permanently changed. It was a great shock to discover that classical concepts, accepted unquestioningly, were inadequate to meet the actual situation, and the shock of this discovery has resulted in a critical attitude toward our whole conceptual structure which must at least in part be permanent. Reflection on the situation after the event shows that it should not have needed the new experimental facts which led to relativity to convince us of the inadequacy of our previous concepts, but that a sufficiently shrewd analysis should have prepared us for at least the possibility of what Einstein did.

Our ideas of nature will change in the course of future experience, but our conceptual approach will remain permanent

Looking now to the future, our ideas of what external nature is will always be subject to change as we gain new experimental knowledge, but there is a part of our attitude to nature which should not be subject to future change, namely that part which rests on the permanent basis of the character of our minds. It is precisely here, in an improved understanding of our mental relations to nature, that the permanent contribution of relativity is to be found. We should now make it our business to understand so thoroughly the character of our permanent mental relations to nature that another change in our attitude, such as that due to Einstein, shall be forever impossible. It was perhaps excusable that a revolution in mental attitude should occur once, because after all physics is a young science, and physicists have been very busy, but it would certainly be a reproach if such a revolution should ever prove necessary again.

NEW KINDS OF EXPERIENCE ALWAYS POSSIBLE

One lesson of the relativity theory is that every theory, however persuasive at present, can be undermined by future experience

The first lesson of our recent experience with relativity is merely an intensification and emphasis of the lesson which all past experience has also taught, namely, that when experiment is pushed into new domains, we must be prepared for new facts, of an entirely different character from those of our former experience. This is taught not only by the discovery of those unsuspected properties of matter moving with high velocities, which inspired the theory of relativity, but also even more emphatically by the new facts in the quantum domain. To a certain extent, of course, the recognition of all this does not involve a change of former attitude; the *fact* has always been for the physicist the one ultimate thing from which there is no appeal, and in the face of which the only possible attitude is a humility almost

religious. The new feature in the present situation is an intensified conviction that in reality new orders of experience do exist, and that we may expect to meet them continually. We have already encountered new phenomena in going to high velocities, and in going to small scales of magnitude: we may similarly expect to find them, for example, in dealing with relations of cosmic magnitudes, or in dealing with the properties of matter of enormous densities, such as is supposed to exist in the stars.

Implied in this recognition of the possibility of new experience beyond our present range, is the recognition that no element of a physical situation, no matter how apparently irrelevant or trivial, may be dismissed as without effect on the final result until proved to be without effect by actual experiment.

Pure empiricism: no a priori principles to be admitted

The attitude of the physicist must therefore be one of pure empiricism. He recognizes no *a priori* principles which determine or limit the possibilities of new experience. Experience is determined only by experience. This practically means that we must give up the demand that all nature be embraced in any formula, either simple or complicated. It may perhaps turn out eventually that as a matter of fact nature can be embraced in a formula, but we must so organize our thinking as not to demand it as a necessity.

THE OPERATIONAL CHARACTER OF CONCEPTS

Einstein's Contribution in Changing Our Attitude Toward Concepts

Recognizing the essential unpredictability of experiment beyond our present range, the physicist, if he is to escape continually revising his attitude, must use in describing and correlating nature concepts of such a character that our present experience does not exact hostages of the future. Now here it seems to me is the greatest contribution of Einstein. Although he himself does not explicitly state or emphasize it, I believe that a study of what he has done will show that he has essentially modified our view of what the concepts useful in physics are and should be. Hitherto many of the concepts of physics have been defined in terms of their properties. An excellent example is afforded by Newton's concept of absolute time. The following quotation from the Scholium in Book I of the *Principia* is illuminating:

Some fundamental elements of physical theory were identified by their properties, with no guarantee that anything with such properties existed in reality. Example: Newton's absolute time

> I do not define Time, Space, Place or Motion, as being well known to all. Only I must observe that the vulgar conceive those quantities under no other notions but from the relation they bear to sensible objects. And thence arise certain prejudices, for the removing of which, it will be convenient to distinguish them into Absolute and Relative, True and Apparent, Mathematical and Common.
> (1) Absolute, True, and Mathematical Time, of itself, and from its own nature flows equably without regard to anything external, and by another name is called Duration.

Now there is no assurance whatever that there exists in nature anything with properties like those assumed in the definition, and physics, when reduced to concepts of this character, becomes as purely an

abstract science and as far removed from reality as the abstract geometry of the mathematicians, built on postulates. It is a task for experiment to discover whether concepts so defined correspond to anything in nature, and we must always be prepared to find that the concepts correspond to nothing or only partially correspond. In particular, if we examine the definition of absolute time in the light of experiment, we find nothing in nature with such properties.

Hence physics became as abstract as mathematics, with the ensuing problem of applicability

The new attitude toward a concept is entirely different. We may illustrate by considering the concept of length: what do we mean by the length of an object? We evidently know what we mean by length if we can tell what the length of any and every object is, and for the physicist nothing more is required. To find the length of an object, we have to perform certain physical operations. The concept of length is therefore fixed when the operations by which length is measured are fixed: that is, the concept of length involves as much as and nothing more than the set of operations by which length is determined. In general, we mean by any concept nothing more than a set of operations; ***the concept is synonymous with the corresponding set of operations.*** If the concept is physical, as of length, the operations are actual physical operations, namely, those by which length is measured; or if the concept is mental, as of mathematical continuity, the operations are mental operations, namely those by which we determine whether a given aggregate of magnitudes is continuous. It is not intended to imply that there is a hard

Every concept should be defined by a set of operations

Different kinds of operations for different kinds of concepts and fast division between physical and mental concepts, or that one kind of concept does not always contain an element of the other; this classification of concept is not important for our future considerations.

We must demand that the set of operations equivalent to any concept be a unique set, for otherwise there are possibilities of ambiguity in practical applications which we cannot admit.

The concept of absolute time is meaningless: we cannot associate with it operations for measuring time

Applying this idea of "concept" to absolute time, we do not understand the meaning of absolute time unless we can tell how to determine the absolute time of any concrete event, *i.e.,* unless we can measure absolute time. Now we merely have to examine any of the possible operations by which we measure time to see that all such operations are relative operations. Therefore the previous statement that absolute time does not exist is replaced by the statement that absolute time is meaningless. And in making this statement we are not saying something new about nature, but are merely bringing to light implications already contained in the physical operations used in measuring time.

This claim is frankly obscure

It is evident that if we adopt this point of view toward concepts, namely that the proper definition of a concept is not in terms of its properties but in terms of actual operations, we need run no danger of having to revise our attitude toward nature. For if experience is always described in terms of experience, there must always be correspondence between experience and our description of it, and we need

We cannot conceptualize reality inaccessible to actual operations

never be embarrassed, as we were in attempting to find in nature the prototype of Newton's absolute time. Furthermore, if we remember that the operations to which a physical concept are equivalent are actual physical operations, the concepts can be defined only in the range of actual experiment, and are undefined and meaningless in regions as yet untouched by experiment. It follows that strictly speaking we cannot make statements at all about regions as yet untouched, and that when we do make such statements, as we inevitably shall, we are making a conventionalized extrapolation, of the looseness of which we must be fully conscious, and the justification of which is in the experiment of the future.

Speculations about such parts of reality will be loose talk

There probably is no statement either in Einstein or other writers that the change described above in the use of "concept" has been self-consciously made, but that such is the case is proved, I believe, by an examination of the way concepts are now handled by Einstein and others. For of course the true meaning of a term is to be found by observing what a man does with it, not by what he says about it. We may show that this is the actual sense in which concept is coming to be used by examining in particular Einstein's treatment of simultaneity.

We discover the meaning of a term by examining the use we make of it

The concept of absolute simultaneity

Before Einstein, the concept of simultaneity was defined in terms of properties. It was a property of two events, when described with respect to their relation in time, that one event was either before the other, or after it, or simultaneous with it. Simultaneity was a property of the two events alone and nothing else; either two events were simultaneous or they were not. The justification for using this term in this way was that it seemed to describe the behavior of actual things. But of course experience then was restricted to a narrow range. When the range of experience was broadened, as by going to high velocities, it was found that the concepts no longer applied, because there was no counterpart in experience for this absolute relation between two events.

New kinds of experience showed the inadequacy of the absolute simultaneity

Einstein now subjected the concept of simultaneity to a critique, which consisted essentially in showing that the operations which enable two events to be described as simultaneous involve measurements on the two events made by an observer, so that "simultaneity" is, therefore, not an absolute property of the two events and nothing else, but must also involve the relation of the events to the observer. Until therefore we have experimental proof to the contrary, we must be prepared to find that the simultaneity of two events depends on their relation to the observer, and in particular on their velocity. Einstein, in thus analyzing what is involved in making a judgment of simultaneity, and in seizing on the act of the observer as the essence of the situation, is actually adopting a new point of view as to what the concepts of physics should be, namely, the operational view.

Relative simulaneity involves relation to observer

Of course Einstein actually went much further than this, and found precisely how the operations for judging simultaneity change when the observer moves, and obtained quantitative expressions for the

effect of the motion of the observer on the relative time of two events. We may notice, parenthetically, that there is much freedom of choice in selecting the exact operations; those which Einstein chose were determined by convenience and simplicity with relation to light beams. Entirely apart from the precise quantitative relations of Einstein's theory, however, the important point for us is that if we had adopted the operational point of view, we would, before the discovery of the actual physical facts, have seen that simultaneity is essentially a relative concept, and would have left room in our thinking for the discovery of such effects as were later found.

Detailed Discussion of the Concept of Length

We may now gain further familiarity with the operational attitude toward a concept and some of its implications by examining from this point of view the concept of length. Our task is to find the operations by which we measure the length of any concrete physical object. We begin with objects of our commonest experience, such as a house or a house lot. What we do is sufficiently indicated by the following rough description. We start with a measuring rod, lay it on the object so that one of its ends coincides with one end of the object, mark on the object the position of the other end of the rod, then move the rod along in a straight line extension of its previous position until the first end coincides with the previous position of the second end, repeat this process as often as we can, and call the length the total num-

How to associate operations with the concept of length

Begin by fixing the measuring rod

ber of times the rod was applied. This procedure, apparently so simple, is in practice exceedingly complicated, and doubtless a full description of all the precautions that must be taken would fill a large treatise. We must, for example, be sure that the temperature of the rod is the standard temperature at which its length is defined, or else we must make a correction for it; or we must correct for the gravitational distortion of the rod if we measure a vertical length; or we must be sure that the rod is not a magnet or is not subject to electrical forces. All these precautions would occur to every physicist. But we must also go further and specify all the details by which the rod is moved from one position to the next on the object—its precise path through space and its velocity and acceleration in getting from one position to another. Practically of course, precautions such as these are not mentioned, but the justification is in our experience that variations of procedure of this kind are without effect on the final result. But we always have to recognize that all our experience is subject to error, and that at some time in the future we may have to specify more carefully the acceleration, for example, of the rod in moving from one position to another, if experimental accuracy should be so increased as to show a measureable effect. In *principle* the operations by which length is measured should be *uniquely* specified. If we have more than one set of operations, we have more than one concept, and strictly there should be a separate name to correspond to each different set of operations.

We must ensure that the rod is free of deformations

Our precautions are always imperfect

Measurement of a moving object

So much for the length of a stationary object, which is complicated enough. Now suppose we have to measure a moving street car. The simplest, and what we may call the "naïve" procedure, is to board the car with our meter stick and repeat the operations we would apply to a stationary body. Notice that this procedure reduces to that already adopted in the limiting case when the velocity of the street car vanishes. But here there may be new questions of detail. How shall we jump on to the car with our stick in hand? Shall we run and jump on from behind, or shall we let it pick us up from in front? Or perhaps does now the material of which the stick is composed make a difference, although previously it did not? All these questions must be answered by experiment. We believe from present evidence that it makes no difference how we jump on to the car, or of what material the rod is made, and that the length of the car found in this way will be the same as if it were at rest. But the experiments are more difficult, and we are not so sure of our conclusions as before. Now there are very obvious limitations to the procedure just given. If the street car is going too fast, we can not board it directly, but must use devices, such as getting on from a moving automobile; and, more important still, there are limitations to the velocity that can be given to street cars or to meter sticks by any practical means in our control, so that the moving bodies which can be measured in this way are restricted to a low range of velocity. If we want to be able to measure the length of bodies mov-

Practical complications may affect our definition

Measuring a fast moving object

ing with higher velocities such as we find existing in nature (stars or cathode particles), we must adopt another definition and other operations for measuring length, which also reduce to the operations already adopted in the static case. This is precisely what Einstein did. Since Einstein's operations were different from our operations above, *his "length" does not mean the same as our "length."* We must accordingly be prepared to find that the length of a moving body measured by the procedure of Einstein is not the same as that above; this of course is the fact, and the transformation formulas of relativity give the precise connection between the two lengths.

Mathematical simplicity is another factor in definition

Einstein's procedure for measuring the length of bodies in motion was dictated not only by the consideration that it must be applicable to bodies with high velocities, but also by mathematical convenience, in that Einstein describes the world mathematically by a system of coördinate geometry, and the "length" of an object is connected simply with quantities in the analytic equations.

Description of Einstein's proposal

It is of interest to describe briefly Einstein's actual operations for measuring the length of a body in motion; it will show how operations which may be simple from a mathematical point of view may appear complicated from a physical viewpoint. The observer who is to measure the length of a moving object must first extend over his entire plane of reference (for simplicity the problem is considered two-dimensional) a system of time coördinates, *i.e.,* at each point of his plane of reference there must be a

Clocks at every point

clock, and all these clocks must be synchronized. At each clock an observer must be situated. Now to find the length of the moving object at a specified instant of time (it is a subject for later investigation to find whether its length is a function of time), the two observers who happen to coincide in position with the two ends of the object at the specified time on their clocks are required to find the distance between their two positions by the procedure for measuring the length of a stationary object, and this distance is by definition the length of the moving object in the given reference system. This procedure for measuring the length of a body in motion hence involves the idea of simultaneity, through the simultaneous position of the two ends of the rod, and we have seen that the operations by which simultaneity are determined are relative, changing when the motion of the system changes. We hence are prepared to find a change in the length of a body when the velocity of the measuring system changes, and this in fact is what happens. The precise numerical dependence is worked out by Einstein, and involves other considerations, in which we are not interested at present.

At each instant the procedure of measuring stationary objects is applied

Involvement of simultaneity entails different lengths at different velocities

The two sorts of length, the naïve one and that of Einstein, have certain features in common. In either case in the limit, as the velocity of the measuring system approaches zero, the operations approach those for measuring the length of a stationary object. This, of course, is a requirement in any good definition, imposed by considerations of convenience, and

The old defintion of length is a limiting case of the new definition it is too obvious a matter to need elaboration. Another feature is that the operations equivalent to either concept both involve the motion of the system, so that we must recognize the possibility that the length of a moving object may be a function of its velocity. It is a matter of experiment, unpredictable until tried, that within the limits of present experimental error the naïve length is not affected by motion, and Einstein's length is.

Length becomes dependent on velocity

Measuring large objects

So far, we have extended the concept of length in only one way beyond the range of ordinary experience, namely to high velocities. The extension may obviously be made in other directions. Let us inquire what are the operations by which we measure the length of a very large object. In practice we probably first meet the desirability of a change of procedure in measuring large pieces of land. Here our procedure depends on measurments with a surveyor's theodolite. This involves extending over the surface of the land a system of coördinates, starting from a base line measured with a tape in the conventional way, sighting on distant points from the extremities of the line, and measuring the angles. Now in this extension we have made one very essential change: the angles between the lines connecting distant points are now angles between beams of light. We assume that a beam of light travels in a straight line. Furthermore, we assume in extending our system of triangulation over the surface of the earth that the geometry of light beams is Euclidean. We do the best we can to check the assumptions, but

Observation of distant point requires assumptions about the path of light and spatial geometry

at most can never get more than a partial check. Thus Gauss[1] checked whether the angles of a large terrestrial triangle add to two right angles and found agreement within experimental error. We now know from the experiments of Michelson[2] that if his measurements had been accurate enough he would not have got a check, but would have had an excess or defect according to the direction in which the beam of light travelled around the triangle with respect to the rotation of the earth. But if the geometry of light beams is Euclidean, then not only must the angles of a triangle add to two right angles, but there are definite relations between the lengths of the sides and the angles, and to check these relations the sides should be measured by the old procedure with a meter stick. Such a check on a large scale has never been attempted, and is not feasible. It seems, then, that our checks on the Euclidean character of optical space are all of restricted character. We have apparently proved that up to a certain scale of magnitude optical space is Euclidean with respect to measures of angle, but this may not necessarily involve that space is also Euclidean with respect to measures of length, so that space need not be completely Euclidean. There is a further most important restriction in that our studies of non-Euclidean geometry have shown that the *percentage* excess of the angles of a non-Euclidean triangle over 180°

Gauss' experiment

The Euclidean character of visual space can only be verified for small scales

[1] C. F. Gauss, Gesammelte Werke, especially vol. IV.

[2] See a discussion of the theory of this experiment by L. Silberstein, Jour. Opt. Soc. Amer. 5, 291-307, 1921.

may depend on the magnitude of the triangle, so that it may well be that we have not detected the non-Euclidean character of space simply because our measurements have not been on a large enough scale.

The concept of length at larger scales is different from the concept of length at smaller scales

We thus see that the concept of length has undergone a very essential change of character even within the range of terrestrial measurements, in that we have substituted for what I may call the tactual concept an optical concept, complicated by an assumption about the nature of our geometry. From a very direct concept we have come to a very indirect concept with a most complicated set of operations. Strictly speaking, length when measured in this way by light beams should be called by another name, since the operations are different. The practical justification for retaining the same name is that within our present experimental limits a numerical difference between the results of the two sorts of operations has not been detected.

Measurement of length at solar and stellar distances

We are still worse off when we make the extension to solar and stellar distances. Here space is entirely optical in character, and we never have an opportunity of even partially comparing tactual with optical space. No direct measures of length have ever been made, nor can we even measure the three angles of a triangle and so check our assumption that the use of Euclidean geometry in extending the concept of space is justified. We never have under observation more than two angles of a triangle, as when we measure the distance of the moon by observation from the two

Measurement at astronomic scales requires assumptions of the laws of mechanics

ends of the earth's diameter. To extend to still greater distance our measures of length, we have to make still further assumptions, such as that inferences from the Newtonian laws of mechanics are valid. The accuracy of our inferences about lengths from such measurements is not high. Astronomy is usually regarded as a science of extraordinarily high accuracy, but its accuracy is very restricted in character, namely to the measurement of angles. It is probably safe to say that no astronomical distance, except perhaps that of the moon, is known with an accuracy greater than 0.1%. When we push our estimates to distances beyond the confines of the solar system in which we are assisted by the laws of mechanics, we are reduced in the first place to measurements of parallax, which at best have a quite inferior accuracy, and which furthermore fail entirely outside a rather restricted range. For greater stellar distances we are driven to other and much rougher estimates, resting for instance on the extension to great distances of connections found within the range of parallax between brightness and spectral type of a star, or on such assumptions as that, because a group of stars looks as if it were all together in space and had a common origin, it actually is so. Thus at greater and greater distances not only does experimental accuracy become less, but the very nature of the operations by which length is to be determined becomes indefinite, so that the distances of the most remote stellar objects as estimated by different observers or by different methods may be very

At still greater distances we no longer have a grip on the set of operations to associate the concept of length with

Whether space as a whole is Euclidean or not is not a question of physics

divergent. A particular consequence of the inaccuracy of the astronomical measures of great distances is that the question of whether large scale space is Euclidean or not is merely academic.

Different lengths for different scales

We thus see that in the extension from terrestrial to great stellar distances the concept of length has changed completely in character. To say that a certain star is 10^5 light years distant is actually and conceptually an entire different *kind* of thing from saying that a certain goal post is 100 meters distant. Because of our conviction that the character of our experience may change when the range of phenomena changes, we feel the importance ot such a question as whether the space of distances of 10^5 light years is Euclidean or not, and are correspondingly dissatisfied that at present there seems no way of giving meaning to it.

Length at small scales

We encounter difficulties similar to those above, and are also compelled to modify our procedures, when we go to small distances. Down to the scale of microscopic dimensions a fairly straightforward extension of the ordinary measuring procedure is sufficient, as when we measure a length in a micrometer eyepiece of a microscope. This is of course a combination of tactual and optical measurements, and certain assumptions, justified as far as possible by experience, have to be made about the behavior of light beams. These assumptions are of a quite different character from those which give us concern on the astronomical scale, because here we meet difficulty from interference effects due to the finite

The role of light beams

scale of the structure of light, and are not concerned with a possible curvature of light beams in the long reaches of space. Apart from the matter of convenience, we might also measure small distances by the tactual method.

As the dimensions become smaller, certain difficulties become increasingly important that were negligible on a larger scale. In carrying out physically the operations equivalent to our concepts, there are a host of practical precautions to be taken which could be explicitly enumerated with difficulty, but of which nevertheless any practical physicist is conscious. Suppose, for example, we measure length tactually by a combination of Johanssen gauges. In piling these together, we must be sure that they are clean, and are thus in actual contact. Particles of mechanical dirt first engage our attention. Then as we go to smaller dimensions we perhaps have to pay attention to adsorbed films of moisture, then at still smaller dimensions to adsorbed films of gas, until finally we have to work in a vacuum, which must be the more nearly complete the smaller the dimensions. About the time that we discover the necessity for a complete vacuum, we discover that the gauges themselves are atomic in structure, that they have no definite boundaries, and therefore no definite length, but that the length is a hazy thing, varying rapidly in time between certain limits. We treat this situation as best we can by taking a time average of the apparent positions of the boundaries, assuming that along with the decrease of dimensions we have acquired a corresponding extravagant increase in nimbleness. But as the dimensions get smaller continually, the difficulties due to this haziness increase indefinitely in percentage effect, and we are eventually driven to give up altogether. We have made the discovery that there are *essential* physical limitations to the operations which defined the concept of length. [We perhaps do not regard the substitution of optical for tactual space on the astronomical scale as compelled by the same sort of physical necessity, because I suppose the possible eventual landing of men in the moon will always be one of the dreams of humanity.] At the same time that we have come to the end of our rope with our Johanssen gauge procedure, our companion with the microscope has been encountering difficulties due to the finite wave length of light; this difficulty he has been able to minimize by using light of progressively shorter wave lengths, but he has eventually had to stop on reaching X-rays. Of course this optical procedure with the microscope is more convenient, and is therefore adopted in practice.

Let us now see what is implied in our concept of length extended to ultramicroscopic dimensions. What, for instance, is the meaning of the statement that the distance between the planes of atoms in a certain crystal is 3×10^{-8} cm.? What we would like to mean is that $1/3 \times 10^{8}$ of these planes piled on top of each other give a thickness of 1 cm.; but of course such a meaning is not the actual one. The actual meaning may be found by examining the operations

by which we arrived at the number 3×10^{-8}. As a matter of fact, 3×10^{-8} was the number obtained by solving a general equation derived from the wave theory of light, into which certain numerical data obtained by experiments with X-rays had been substituted. Thus not only has the character of the concept of length changed from tactual to optical, but we have gone much further in committing ourselves to a definite optical theory. If this were the whole story, we would be most uncomfortable with respect to this branch of physics, because we are so uncertain of the correctness of our optical theories, but actually a number of checks can be applied which greatly restore our confidence. For instance, from the density of the crystal and the grating space, the weight of the individual atoms may be computed, and these weights may then be combined with measurements of the dimensions of other sorts of crystal into which the same atoms enter to give values of the densities of these crystals, which may be checked against experiment. All such checks have succeeded within limits of accuracy which are fairly high. It is important to notice that, in spite of the checks, the character of the concept is changing, and begins to involve such things as the equations of optics and the assumption of the conservation of mass.

Measurement at subatomic level

We are not content, however, to stop with dimensions of atomic order, but have to push on to the electron with a diameter of the order of 10^{-13} cm. What is the possible meaning of the statement that the diameter of an electron is 10^{-13} cm.? Again the only an-

Requires assumptions about electromagnetism at subatomic level

Assignment of space coordinates depends on equations describing the elecrtromagnetic field, but the equations can only be solved after the assignment of coordinates

swer is found by examining the operations by which the number 10^{-13} was obtained. This number came by solving certain equations derived from the field equations of electrodynamics, into which certain numerical data obtained by experiment had been substituted. The concept of length has therefore now been so modified as to include that theory of electricity embodied in the field equations, and, most important, assumes the correctness of extending these equations from the dimensions in which they may be verified experimentally into a region in which their correctness is one of the most important and problematical of present-day questions in physics. To find whether the field equations are correct on a small scale, we must verify the relations demanded by the equations between the electric and magnetic forces and the space coördinates, to determine which involves measurement of lengths. But if these space coördinates cannot be given an independent meaning apart from the equations, not only is the attempted verification of the equations impossible, but the question itself is meaningless. If we stick to the concept of length by itself, we are landed in a vicious circle. As a matter of fact, the concept of length disappears as an independent thing, and fuses in a complicated way with other concepts, all of which are themselves altered thereby, with the result that the total number of concepts used in describing nature at this level is reduced in number. A precise analysis of the situation is difficult, and I suppose has never been attempted, but the general character of the situation is

The question of geometry at subatomic level is meaningless

evident. Until at least a partial analysis is attempted, I do not see how any meaning can be attached to such questions as whether space is Euclidean in the small scale.

It is interesting to observe that any increased accuracy in knowledge of large scale phenomena must, as far as we now can see, arise from an increase in the accuracy of measurement of small things, that is, in the measurement of small angles or the analysis of minute differences of wave lengths in the spectra. To know the very large takes us into the same field of experiment as to know the very small, so that operationally the large and the small have features in common.

A change in associated operations entails a change of concept

This somewhat detailed analysis of the concept of length brings out features common to all our concepts. If we deal with phenomena outside the domain in which we originally defined our concepts, we may find physical hindrances to performing the operations of the original definition, so that the original operations have to be replaced by others. These new operations are, of course, to be so chosen that they give, within experimental error, the same numerical results in the domain in which the two sets of operations may be both applied; but we must recognize in principle that in changing the operations we have really changed the concept, and that to use the same name for these different concepts over the entire range is dictated only by considerations of convenience, which may sometimes prove to have been purchased at too high a price in terms of unambiguity.

We must always be prepared some day to find that an increase in experimental accuracy may show that the two different sets of operations which give the same results in the more ordinary part of the domain of experience, lead to measurably different results in the more unfamiliar parts of the domain. We must remain aware of these joints in our conceptual structure if we hope to render unnecessary the services of the unborn Einsteins.

Reduction in the number of concepts at the boundaries of experience

The second feature common to all concepts brought out by the detailed discussion of length is that, as we approach the experimentally attainable limit, concepts lose their individuality, fuse together, and become fewer in number, as we have seen that at dimensions of the order of the diameter of an electron the concepts of length and the electric field vectors fuse into an amorphous whole. Not only does nature as experienced by us become different in character on its horizons, but it becomes simpler, and therefore our concepts, which are the building stones of our descriptions, become fewer in number. This seems to be an entirely natural state of affairs. How the number of concepts is often kept formally the same as we approach the horizon will be discussed later in special cases.

A precise analysis of our conceptual structure has never been attempted, except perhaps in very restricted domains, and it seems to me that there is room here for much important future work. Such an analysis is not to be attempted in this essay, but only some of the more important qualitative aspects are to

be pointed out. It will never be possible to give a clean-cut logical analysis of the conceptual situation, for the nature of our concepts, according to our operational point of view, is the same as the nature of experimental knowledge, which is often hazy. Thus in the transition regions where nature is getting simpler and the number of operationally independent concepts changes, a certain haziness is inevitable, for the actual change in our conceptual structure in these transition regions is continuous, corresponding to the continuity of our experimental knowledge, whereas formally the number of concepts should be an integer.

The Relative Character of Knowledge

Two other consequences of the operational point of view must now be examined. First is the consequence that all our knowledge is relative. This may be understood in a general or a more particular sense. The general sense is illustrated in Haldane's book on the *Reign of Relativity*. Relativity in the general sense is the merest truism if the operational definition of concept is accepted, for experience is described in terms of concepts, and since our concepts are constructed of operations, all our knowledge must unescapably be relative to the operations selected. But knowledge is also relative in a narrower sense, as when we say there is no such thing as absolute rest (or motion) or absolute size, but rest and size are relative terms. Conclusions of this kind are involved in the specific character of the operations in

Our knowledge rests on concepts that are defined in reference to an observer

Concepts are defined in reference to measuring instruments terms of which rest or size are defined. An examination of the operations by which we determine whether a body is at rest or in motion shows that the operations are relative operations: rest or motion is determined with respect to some other body selected as the standard. In saying that there is no such thing as absolute rest or motion we are not making a statement about nature in the sense that might be supposed, but we are merely making a statement about the character of our descriptive processes. Similarly with regard to size: examination of the operations of the measuring process shows that size is measured relative to the fundamental measuring rod.

The "absolute" therefore disappears in the original meaning of the word. But the "absolute" may usefully return with an altered meaning, and we may say that a thing has absolute properties if the numerical magnitude is the same when measured with the same formal procedure by all observers. Whether a given property is absolute or not can be determined only by experiment, landing us in the paradoxical position that the absolute is absolute only relative to experiment. In some cases, the most superficial observation shows that a property is not absolute, as, for example, it is at once obvious that measured velocity changes with the motion of the observer. But in other cases the decision is more difficult. Thus Michelson thought he had an absolute procedure for measuring length, by referring to the wave length of

the red cadmium line as standard;[1] it required difficult and accurate experiment to show that this length varies with the motion of the observer. Even then, by changing the definition of the length of a moving object, we believe that length might be made to reassume its desired absolute character.

To stop the discussion at this point might leave the impression that this observation of the relative character of knowledge is of only a very tenuous and academic interest, since it appears to be concerned mostly with the character of our descriptive processes, and to say little about external nature. [What this means we leave to the metaphysician to decide.] But I believe there is a deeper significance to all this. It must be remembered that all our argument starts with the concepts as given. Now these concepts involve physical operations; in the discovery of what operations may be usefully employed in describing nature is buried almost all physical experience. In erecting our structure of physical science, we are building on the work of all the ages. There is then this purely physical significance in the statement that all motion is relative, namely that no operations of measuring motion have been found to be useful in describing simply the behavior of nature which are not operations relative to a single observer; in making this statement we are stating something about nature. It takes an enormous amount of real physical experi-

[1] A. A. Michelson, Light Waves and Their Uses, University of Chicago Press, 1903, Chap. V.

ence to discover relations of this sort. The discovery that the number obtained by counting the number of times a stick may be applied to an object can be simply used in describing natural phenomena was one of the most important and fundamental discoveries ever made by man.

Meaningless Questions

Certain claims are meaningless, since they employ non-operationally defined concepts

Another consequence of the operational character of our concepts, almost a corollary of that considered above, is that it is quite possible, nay even disquietingly easy, to invent expressions or to ask questions that are meaningless. It constitutes a great advance in our critical attitude toward nature to realize that a great many of the questions that we uncritically ask are without meaning. If a specific question has meaning, it must be possible to find operations by which an answer may be given to it. It will be found in many cases that the operations cannot exist, and the question therefore has no meaning. For instance, it means nothing to ask whether a star is at rest or not. Another example is a question proposed by Clifford, namely, whether it is not possible that as the solar system moves from one part of space to another the absolute scale of magnitude may be changing, but in such a way as to affect all things equally, so that the change of scale can never be detected. An examination of the operations by which length is measured in terms of measuring rods shows that the operations do not exist (because of the nature of our definition of length) for answering the question.

Examples based on the (meaningless) idea of absolute space

The question can be given meaning only from the point of view of some imaginary superior being watching from an external point of vantage. But the operations by which such a being measures length are different from the operations of our definition of length, so that the question acquires meaning only by changing the significance of our terms—in the original sense the question means nothing.

The significance of meaningless questions

To state that a certain question about nature is meaningless is to make a significant statement about nature itself, because the fundamental operations are determined by nature, and to state that nature cannot be described in terms of certain operations is a significant statement.

Different layers of meaning

It must be recognized, however, that there is a sense in which no serious question is entirely without meaning, because doubtless the questioner had in mind some intention in asking the question. But to give meaning in this sense to a question, one must inquire into the meaning of the concepts as used by the questioner, and it will often be found that these concepts can be defined only in terms of fictitious properties, as Newton's absolute time was defined by its properties, so that the meaning to be ascribed to the question in this way has no connection with reality. I believe that it will enable us to make more significant and interesting statements, and therefore will be more useful, to adopt exclusively the operational view, and so admit the possibility of questions entirely without meaning.

This matter of meaningless questions is a very subtle thing which may poison much more of our thought than that dealing with purely physical phenomena. I believe that many of the questions asked about social and philosophical subjects will be found to be meaningless when examined from the point of view of operations. It would doubtless conduce greatly to clarity of thought if the operational mode of thinking were adopted in all fields of inquiry as well as in the physical. Just as in the physical domain, so in other domains, one is making a significant statement about his subject in stating that a certain question is meaningless.

An interesting (and difficult) list of possibly meaningless questions

In order to emphasize this matter of meaningless questions, I give here a list of questions, with which the reader may amuse himself by finding whether they have meaning or not.

(1) Was there ever a time when matter did not exist?
(2) May time have a beginning or an end?
(3) Why does time flow?
(4) May space be bounded?
(5) May space or time be discontinuous?
(6) May space have a fourth dimension, not directly detectible, but given indirectly by inference?
(7) Are there parts of nature forever beyond our detection?
(8) Is the sensation which I call blue really the *same* as that which my neighbor calls blue? Is it possible that a blue object may arouse in him the same sensation that a red object does in me and *vice versa?*
(9) May there be missing integers in the series of natural numbers as we know them?

(10) Is a universe possible in which $2+2 \neq 4$?
(11) Why does negative electricity attract positive?
(12) Why does nature obey laws?
(13) Is a universe possible in which the laws are different?
(14) If one part of our universe could be *completely* isolated from the rest, would it continue to obey the same laws?
(15) Can we be sure that our logical processes are valid?

General Comments on the Operational Point of View

A call for adoption of operationalism

To adopt the operational point of view involves much more than a mere restriction of the sense in which we understand "concept," but means a far-reaching change in all our habits of thought, in that we shall no longer permit ourselves to use as tools in our thinking concepts of which we cannot give an adequate account in terms of operations. In some respects thinking becomes simpler, because certain old generalizations and idealizations become incapable of use; for instance, many of the speculations of the early natural philosophers become simply unreadable. In other respects, however, thinking becomes much more difficult, because the operational implications of a concept are often very involved. For example, it is most difficult to grasp adequately all that is contained in the apparently simple concept of "time," and requires the continual correction of mental tendencies which we have long unquestioningly accepted.

Operational thinking will at first prove to be an unsocial virtue; one will find oneself perpetually unable to understand the simplest conversation of one's friends, and will make oneself universally unpopular by demanding the meaning of apparently the simplest terms of every argument. Possibly after every one has schooled himself to this better way, there will remain a permanent unsocial tendency, because doubtless much of our present conversation will then become unnecessary. The socially optimistic may venture to hope, however, that the ultimate effect will be to release one's energies for more stimulating and interesting interchange of ideas.

Not only will operational thinking reform the social art of conversation, but all our social relations will be liable to reform. Let any one examine in operational terms any popular present-day discussion of religious or moral questions to realize the magnitude of the reformation awaiting us. Wherever we temporize or compromise in applying our theories of conduct to practical life we may suspect a failure of operational thinking.