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HERMANN VON HELMHOLTZ

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NEWLY TRANSLATED BY MALCOLM F. LOWE  
EDITED, WITH AN INTRODUCTION AND BIBLIOGRAPHY, BY  
ROBERT S. COHEN AND YEHUDA ELKANA



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CHAPTER IV

THE FACTS IN PERCEPTION<sup>1</sup>

Address given during the anniversary celebrations of the  
Friedrich Wilhelm University in Berlin, in 1878; reprinted  
in *Vorträge und Reden*, vol. II, pp. 215–247, 387–406.

My distinguished audience !

Today on the birthday of the founder of our university, the sorely-tried King Friedrich Wilhelm III, we celebrate the anniversary of its foundation. The year of its foundation, 1810, fell in the period of the greatest external stress upon our country. A considerable part of its territory had been lost, the land was exhausted from the preceding war and the enemy occupation. The martial pride which had remained with it from the times of the great elector and the great king had been deeply humiliated. And yet this same period now seems to us, when we glance backwards, to have been so rich in possessions of a spiritual kind, in inspiration, energy, ideal hopes and creative thoughts, that we might, despite the relatively brilliant external situation of our country and nation today, look back upon it almost with envy. If in that distressing situation the king's first thought was of founding the university before other material claims, if he then staked throne and life so as to entrust himself to the resolute inspiration of the nation in the struggle against the conqueror, this all shows how deeply within him too, the simple man disinclined to lively expressions of feeling, acted a trust in the spiritual powers of his people.

At that time Germany could point to a magnificent series of praiseworthy names in both art and science, names whose bearers are in part to be counted amongst the greatest of all times and peoples in the history of human culture.

Goethe was alive and so was Beethoven; Schiller, Kant, Herder and Haydn had survived the first years of the century. Wilhelm von Humboldt was outlining the new science of comparative linguistics; Niebuhr, Fr. Aug. Wolf and Savigny were teaching how to permeate ancient history, poetry and law with living understanding; Schleiermacher was seeking a profound understanding of the spiritual content of religion. Joh. Gottlieb Fichte, the second rector of our university, the powerful and fearless public speaker, was carrying his audience away with the stream of his moral inspiration and the bold intellectual flight of his idealism.

*The state of  
German  
intellectual life  
at the time of  
Napoleonic  
wars (1810).  
Can be skipped.*

Even the aberrations of this mentality, which express themselves in the easily recognizable weaknesses of romanticism, have something attractive compared with dry, calculating egoism. One marvelled at oneself in the fine feelings in which one knew how to revel, one sought to develop the art of having such feelings. One thought oneself allowed to admire fantasy all the more as a creative force, the more it had freed itself from the rules of the understanding. Much vanity lay hidden in this, but all the same a vanity of enthusiasm for high ideals.

The older ones amongst us still knew the men of that period, who had once entered the army as the first volunteers, always ready to immerse themselves in the discussion of metaphysical problems, well-read in the works of Germany's great poets, men who still glowed with rage when talking of the first Napoleon, but with rapture and pride when of deeds in the war of liberation.

How things have changed! We may well exclaim thus with amazement in a period when a cynical contempt for every ideal possession of humankind is propagated, on the streets and in the press, and has reached its peak in two revolting crimes<sup>†</sup>, which were obviously only aimed at the head of our emperor because in him was united everything that humanity, up to now, has regarded as worthy of veneration and gratitude.

We must almost make an effort to recall that only eight years have passed since the great hour, when at the call of the same monarch every rank of our people, without hesitation and filled with self-sacrificing and inspired patriotism, went into a dangerous war against an opponent whose might and valour were not unknown to us. We must almost make an effort to take note of the wide extent to which the endeavours, political and humane, to give the poorer ranks too of our people an existence less troubled and more worthy of human beings, have captured the activity and thoughts of the educated classes. Or to think how much their lot in material and legal respects has actually been improved.

The nature of mankind seems simply to be such that next to much light one can always find much shadow. Political freedom initially gives the vulgar motives a greater licence to reveal themselves and to embolden each other, as long as they are not faced with a public opinion ready to offer energetic opposition. Even in the years before the war of liberation, when Fichte was preaching sermons calling upon his generation to re-

<sup>†</sup> [In 1878, two attempts were made to assassinate the emperor.]

pent, these elements were not lacking. He depicts conditions and sentiments as ruling which recall the worst of our times. "The present age adopts in its basic principle a stance of haughtily looking down upon those who, from a dream of virtue, let themselves be torn away from pleasures, and rejoicing in the thought that one must get beyond such things, and not at all be imposed upon in this manner."\* The only pleasure, going beyond the purely sensuous, which he concedes to be known to the representatives of that age, is what he calls "delighting in one's own artfulness". And yet, in this same period, there was being prepared a mighty upswing which belongs to the most glorious events in our history.

Although we therefore need not regard our period as beyond hope, we should surely not soothe ourselves too easily with the consolation that things were indeed not better in other times than now. It is nevertheless advisable, when such dubious processes are going on, that each person should make a review – in the sphere given him to work in and which he knows – of the situation of the work towards the eternal goals of mankind: whether they are being kept in view, whether one has got nearer to them. In the youthful days of our university science too was youthfully bold and strong in hopes, its view was directed pre-eminently towards the highest goals. Although these were not to be reached so easily as that generation hoped, although it also emerged that long drawn out particular labours had to prepare the path towards them, so that initially the nature itself of the tasks demanded another kind of work – less enthusiastic, less immediately directed towards the ideal goals – it would still doubtless be pernicious should our generation have lost sight of the eternal ideals of mankind, over and above subordinate and practically useful tasks.

In that period, the fundamental problem placed at the beginning of all science was the problem of epistemology: "What is true in our intuition and thought?"<sup>2</sup> In what sense do our representations correspond to actuality?"<sup>3</sup> Philosophy and natural science encounter this problem from two opposite sides, it is a task common to both.

The former, which considers the mental side, seeks to separate out from our knowledge and representation what originates from the influ-

*The question of  
correspondence  
between our  
thought and  
reality  
formulated*

\* Fichte, *Werke*, vol. VII, p. 40.

ences of the corporeal world, in order to set forth unalloyed what belongs to the mind's own workings. Natural science, on the contrary, seeks to separate off that which is definition, symbolism, representational form or hypothesis, in order to have left over unalloyed what belongs to the world of actuality<sup>†</sup> whose laws it seeks. Both seek to execute the same separation, although each is interested in a different part of what is separated<sup>4</sup>. In the theory of sense perceptions, and in investigations into the fundamental principles of geometry, mechanics and physics, even the enquirer into nature cannot evade these questions. As my own studies have frequently entered both domains, I want to try to give you a survey of what has been done in this direction on the part of enquiry into nature.

Naturally, in the last analysis the laws of thought are no different in the man enquiring into nature from what they are in the man who philosophises. In all cases where the facts of daily experience – whose profusion is after all already very great – sufficed to give a percipient thinker, with an unconstrained feeling for truth, in some measure enough material for a correct judgement, the enquirer into nature must satisfy himself with acknowledging, that the methodically completed gathering of empirical facts simply confirms the result gained previously. But there also occur cases of the contrary kind. Such cases will justify the fact – if it needs to be justified at all – that in what follows the questions concerned are not everywhere given new answers, but to a great extent ones given long ago are repeated. Indeed, often enough even an old concept, measured against new facts, gets a more vivid illumination and a new look.

Shortly before the beginning of the present century, Kant had developed the doctrine of forms of intuiting and thinking given prior to all experience<sup>5</sup> – or (as he therefore termed them) “*transcendental*”<sup>6</sup> forms of intuiting and thinking – into which forms any content we may represent must necessarily be absorbed, if this content is to become a representation. Regarding the qualities of sensation, Locke had already established a claim for the share which our corporeal and mental make-up has in the manner in which things appear to us<sup>7</sup>. In this direction, investigations into the physiology of the senses, which were in particular completed and critically sifted by Johannes Müller and then summarized by him in the law of *specific energies of sensory nerves*, have now brought

<sup>†</sup> [‘Wirklichkeit’; ‘wirklich’ has been translated consistently as ‘actual’ rather than ‘real’, because below Helmholtz expressly distinguishes it from ‘reell’.]

the fullest confirmation, one can almost say to an unexpected degree. At the same time, they have thereby portrayed and made intuitive, in a very decisive and palpable manner, the essence and significance of such a subjective form, given in advance, of sensation. This theme has already often been discussed, for which reason my presentation of it today can be brief.

There occur two distinct degrees of difference between the various kinds of sensation. The more deeply incisive difference is that between sensations belonging to different senses, such as between blue, sweet, warm, highpitched: I have permitted myself to term this a difference in the *modality* of sensation. It is so incisive as to exclude any transition from the one to the other, any relationship of greater or lesser similarity. One cannot at all ask whether e.g. sweet is more similar to blue or to red. On the other hand, the second kind of difference – the less incisive – is that between different sensations of the same sense: I restrict the term a difference of *quality* to this difference alone. Fichte groups together these qualities of a single sense as a quality range<sup>†</sup>, and terms a *difference of quality ranges* what I just called a difference of modality. Within each such range, transition and comparison are possible. We can make the transition from blue through violet and crimson into scarlet, and e.g. declare yellow to be more similar to orange than to blue.

What physiological investigations now show is that the deeply incisive difference does not depend, in any manner whatsoever, upon the kind of external impression whereby the sensation is excited, but is determined alone and exclusively by the sensory nerve upon which the impression impinges. Excitation of the optic nerve produces only light sensations, no matter whether objective light – i.e. aether vibrations – impinges upon it, or an electric current which we pass through the eye, or pressure on the eyeball, or straining of the nerve stem during rapid changes of the direction of vision. The sensation arising through the latter influences is so similar to that of objective light, that people for a long time believed in light actually developing in the eye. Johannes Müller showed that such a development does not on any account take place, that the sensation of light was indeed only there because the optic nerve was excited<sup>8</sup>.

Just as on the one hand each sensory nerve, excited by however so

<sup>†</sup> [‘Qualitätenkreis’]

*Modalities of sensations*

*Physiological analysis of sensations*

*The project of natural science*

*How science can answer epistemological questions*

"Aether  
vibrations"

many influences, always gives only sensations from the quality range proper to itself, so on the other hand are produced by the same external influences – when they impinge upon different sensory nerves – the most varied kinds of sensation, these always being taken from the quality range of the nerve concerned. The same aether vibrations as are felt by the eye as light, are felt by the skin as heat. The same air vibrations as are felt by the skin as a quivering motion, are felt by the ear as a note. Here the difference in kind of the impression is moreover so great, that physicists felt at ease with the idea that agents as apparently different as light and radiant heat are alike in kind, and in part identical, only after the complete likeness in kind of their physical behaviour had been established, by laborious experimental investigations in every direction.

But even within the quality range of each individual sense, where the kind of object exerting an influence at least codetermines the quality of the produced sensation, there still occur the most unexpected incongruities. In this respect, the comparison of eye and ear is instructive. For the objects of both – light and sound – are oscillatory motions<sup>9</sup>, each of which excites different sensations according to the rapidity of vibration: in the eye different colours, in the ear different pitches.

Sight and  
hearing  
contrasted

If we allow ourselves, for the sake of greater perspicuity, to refer to the frequency relationships of light in terms of the musical intervals formed by corresponding tone frequencies, then the result is as follows: the ear is sensitive to some ten octaves of different tones, the eye only to a sixth<sup>†</sup>, although the frequencies lying beyond these limits occur for both sound and light, and can be demonstrated physically. The eye has in its short scale only three mutually distinct basic sensations, out of which all of its qualities are composed by addition, namely red, green and bluish violet. These mix in sensation without interfering with one another<sup>10</sup>. The ear, on the other hand, distinguishes between an enormous number of tones of different pitches. No two chords composed out of different tones ring alike, while yet with the eye precisely the analogue of this is the case. For a white which looks alike can be produced with red and greenish blue from the spectrum, with yellow and ultramarine, with greenish yellow and violet, [with green, red and violet,]<sup>††</sup> or with

<sup>†</sup> [i.e. the interval having this name, not a sixth of an octave.]

<sup>††</sup> [The concluding words of the sentence indicate that Hertz and Schlick omitted this phrase by mistake.]

any two or three – or with all – of these mixtures together. Were the situation alike with the ear, the consonance of *C* and *F* would sound like that of *D* and *G*, *E* and *A*, or *C*, *D*, *E*, *F*, *G* and *A*, etc. And – what is notable as regards the objective significance of colour – apart from the effect on the eye, one has not been able to detect a single physical connexion in which light which looked alike was regularly alike in value.

The whole foundation, finally, of the musical effect of consonance and dissonance depends upon the peculiar phenomenon of beats. The basis of these is a rapid alternation in intensity of tone, which arises from the fact that two tones almost alike in pitch alternately interact with their phases alike and opposed, and correspondingly excite now strong and now weak vibrations in a resonating body. The physical phenomenon might equally well occur through the interaction of two light-wave trains as through the interaction of two sound-wave trains. But the nerve must firstly be capable of being affected by both wave trains, and it must secondly be able to follow quickly enough the alternation of strong and weak intensity. The auditory nerve is markedly superior in the latter respect to the optic nerve. At the same time, each fibre of the auditory nerve is sensitive only to tones from a narrow interval of the scale, so that only tones situated quite near to each other in it can interact at all. Ones far from each other cannot interact, or not directly. When they do, this originates from accompanying overtones or combination tones. There therefore occurs with the ear this difference between resounding and non-resounding intervals, i.e. between consonance and dissonance. Each fibre of the optic nerve, on the other hand, is sensitive throughout the whole spectrum, although with different strength in different parts. Could the optic nerve at all follow in sensation the enormously rapid beats of light oscillations, then every mixed colour would act as a dissonance<sup>11</sup>.

Perception of  
music

You can see how all these differences in the manner of action of light and sound are conditioned by the way in which the nervous apparatus reacts to them.

Summary

Our sensations are indeed effects produced in our organs by external causes<sup>12</sup>; and how such an effect expresses itself<sup>13</sup> naturally depends quite essentially upon the kind of apparatus upon which the effect is produced. Inasmuch as the quality of our sensation gives us a report of what is peculiar to the external influence by which it is excited, it may

No images in  
perception



count as a symbol of it, but not as an *image*. For from an image one requires some kind of likeness with the object of which it is an image – from a statue likeness of form, from a drawing likeness of perspective projection in the visual field, from a painting likeness of colours as well. But a sign need not have any kind of similarity at all with what it is the sign of. The relation between the two of them is restricted to the fact that like objects exerting an influence under like circumstances evoke like signs, and that therefore unlike signs always correspond to unlike influences.

To popular opinion, which accepts in good faith that the images which our senses give us of things are wholly true<sup>14</sup>, this residue of similarity acknowledged by us may seem very trivial. In fact it is not trivial. For with it one can still achieve something of the very greatest importance, namely forming an image of lawfulness in the processes of the actual world<sup>15</sup>. Every law of nature asserts that upon preconditions alike in a certain respect, there always follow consequences which are alike in a certain other respect. Since like things are indicated in our world of sensations by like signs, an equally regular sequence will also correspond in the domain of our sensations to the sequence of like effects by law of nature upon like causes.

If berries of a certain kind in ripening develop at the same time a red pigment and sugar, then a red colour and a sweet taste will always be found together in our sensation for berries of this type.

Thus although our sensations, as regards their quality, are only *signs* whose particular character depends wholly upon our own makeup, they are still not to be dismissed as a mere semblance, but they are precisely signs of *something*, be it something existing or happening, and – what is most important – they can form for us an image of the *law* of this thing which is happening.

So physiology too acknowledges the qualities of sensation to be a mere form of intuition<sup>16</sup>. But Kant went further. He spoke not only of the qualities of sensations as given by the peculiarities of our intuitive faculty, but also of space and time, since we cannot perceive anything in the external world without its happening at a specific time and being situated at a specific place. Specification in time is even an attribute of every internal perception as well. He therefore termed time the given and necessary *transcendental form of inner intuition*, and space the corre-

sponding form of *outer intuition*. Thus Kant considers spatial specifications too as belonging as little to the world of the actual – or to ‘the thing in itself’ – as the colours which we see are attributes of bodies in themselves, but [which]<sup>†</sup> are introduced by our eye into them.

Even here the approach of natural science can take the same path, up to a certain limit. Suppose we namely ask whether there is a common characteristic, perceivable in immediate sensation, whereby every perception relating to objects in space is characterized for us. Then we in fact find such a characteristic in the circumstance that motion of our body places us in different spatial relations to the perceived objects, and thereby also alters the impression made by them upon us. But the impulse to motion, which we give through an innervation of our motor nerves, is something immediately perceivable<sup>17</sup>. That we do something, when we give such an impulse, is felt by us. What we do, we do not know in an immediate manner. Only physiology teaches us that we put into an excited state – or *innervate* – the motor nerves, that their stimulation is passed on to the muscles, that these consequently contract and move the limbs. Yet all the same we know, even without scientific study, which perceivable effect follows each of the various innervations that we are able to initiate.

That we learn it by frequently repeated attempts and observations, may be demonstrated with assurance in a long series of cases. We can learn even as adults to find the innervations needed for pronouncing the letters of a foreign language, or for a particular kind of voice production in singing. We can learn innervations for moving our ears, for squinting with our eyes inwards or outwards, or even upwards and downwards, and so on. The difficulty in performing such things consists only in our having to seek, by making attempts, to find the as yet unknown innervations needed for such previously unexecuted movements. We ourselves, moreover, know of these impulses in no other form, and through no other definable feature, than precisely the fact that they produce the intended observable effect. Thus this effect also alone serves to distinguish the various impulses in our representation<sup>18</sup>.

Now when we give impulses of this sort (turning our gaze, moving our hands, going back and forth), we find that the sensations belonging to

<sup>†</sup> [Something is wrong with Helmholtz' syntax here, but the sense is obvious.]

Only signs of  
external objects

Sign theory  
elaborated

Kant's "pure  
intuition" of  
space and time

Physiological  
analysis of  
spatial intuition

Innervations

certain quality ranges (namely those relating to spatial objects) can thereby be altered; other psychic states of which we are conscious – memories, intentions, wishes, moods – cannot be altered at all. A thoroughgoing difference between the former and the latter is thereby laid down in immediate perception.

*Spatial intuition arises through an alteration during movement*

Thus if we desire to call the relationship which we alter in an immediate manner by the impulses of our will – what kind of relationship this is might moreover be still quite unknown to us – a *spatial* one, then perceptions of *psychic* activities do not enter into such a relationship at all. But probably all sensations of the outer senses must proceed subject to some kind of innervation or another, i.e. have some spatial specification<sup>19</sup>. In this case space will also appear to us – imbued with the qualities of our sensations of movement – in a sensory manner, as that through which we move, through which we can gaze forth. Spatial intuition would therefore be in this sense a subjective *form of intuition*, like the sensory qualities red, sweet and cold<sup>20</sup>. Naturally, the sense of this would just as little be mere semblance for the former as for the latter, the place specified for a specific individual object is no mere *semblance*<sup>21</sup>.

*Our claims are in agreement with Kant!*

From this point of view, however, space would appear as the *necessary* form of outer intuition, because precisely what we perceive as having some spatial specification comprises for us the external world. We comprehend as the world of inner intuition, as the world of self-consciousness, that in which no spatial relation is to be perceived<sup>22</sup>.

And space would be a *given form* of intuition, possessed *prior to all experience*, to the extent that its perception were connected with the possibility of motor impulses of the will the mental and corporeal capacity for which had to be given to us, by our makeup<sup>23</sup>, before we could have spatial intuition<sup>24</sup>.

*See Schlick's notes 22–24*

It will hardly be a matter for doubt, that the characteristic which we have discussed, of altering during movement, is an attribute of all perceptions relating to spatial objects\*. The question will need to be answered, on the other hand, as to whether every specific peculiarity of our spatial intuition is now to be derived from this source. To this end we

<sup>†</sup> [‘räumlich bestimmt sein’, i.e. any such sensation has a feature which can be altered by our moving.]

\* On the localization of sensations of internal organs, see Appendix I to this paper.

must consider what can be attained with the aid of the features of perception which have so far been discussed.

Let us try to put ourselves back into the position of a man without any experience. We must assume, in order to begin without spatial intuition, that such a man knows even the effects of his innervations only to the extent that he has learnt how, by remission of a first innervation or by execution of a second counterimpulse, he can put himself back into the state from which he has removed himself by the first impulse. As this mutual self-cancellation of different innervations is wholly independent of what is thereby perceived, the observer can find out, without yet having previously gained any understanding of the external world, how he has to do this.

Let the situation of the observer initially be that he is faced with an environment of objects at rest. This will make itself known to him in the first place by the fact, that as long as he gives no motor impulse his sensations remain unaltered. If he gives such an impulse (e.g. if he moves his eyes or hands, or steps forward), the sensations alter; and if he then, by remission or the appropriate counterimpulse, returns to the earlier state, all his sensations will again be the earlier ones<sup>25</sup>.

Let us call the whole group of sensation aggregates which can be brought about during the period of time under discussion, by a certain specific and limited group of impulses of the will, the *presentables* for that period; and call *present*, on the other hand, the sensation aggregate from this group which happens to be being perceived. Then our observer is tied at this time to a certain range of presentables, but any individual one of which he can make present at any moment he wishes by executing the relevant movement. Each individual presentable from this group thereby appears to him as *enduring at every moment* of this period of time. He has observed it at every individual moment that he wanted to. The assertion that he would have been able to observe it also at any other intervening moment that he might have wanted to, is to be regarded as an inductive inference, drawn from the case of every moment at which a successful attempt was made to that of every moment whatsoever in the relevant period of time. Therefore, the representation of an *enduring existence of different things at the same time one beside another* can in this manner be acquired.

‘One beside another’ is a spatial description. But it is justified, since

*Physiological derivation of the peculiarities of spatial experience*

*Derivation of spatial coexistence*

we have defined as 'spatial' the relationship altered by impulses of the will. One does not yet need to think of substantial things as what are here supposed to exist one beside another. 'To the right it is bright, to the left it is dark, in front there lies resistance but not behind' could for example be said at this stage of knowledge, with right and left being only names for certain eye movements, in front and behind for certain movements of the hands.

*Derivation of  
spatial  
objecthood*

Now at other times the range of presentables, for the same group of impulses of the will, is going to be a different one. This range, with the individual which it contains, will thereby confront us as something given, as an 'objectum' <sup>†</sup>. Those alterations which we can produce and revoke by conscious impulses of the will, are distinct from ones which are not consequences of such impulses and cannot be eliminated by them. The latter specification is negative. Fichte's appropriate expression for this is that the 'I' is faced with a 'not-I' which exacts recognition <sup>26</sup>.

*Tactile  
sensations can  
give rise to  
spatial  
intuition*

In asking about the empirical conditions under which spatial intuition develops, we must in these considerations take account chiefly of touch, since the blind can develop spatial intuition <sup>27</sup> completely without the help of sight. Although for them space will not turn out to be filled up with objects in such richness and detail as for sighted persons, it yet seems most highly improbable that the foundations of spatial intuition for the two classes of person should be wholly different. If we ourselves attempt to make observations by touching, in the dark or with our eyes closed, then we may very well touch with one finger – or even with a pencil held in the hand like the surgeon with his probe – and still ascertain, in detail and with assurance, the corporeal form of the object present.

When wanting to find our way in the dark, we usually feel over larger objects with five or ten finger-tips simultaneously. We then obtain five to ten times as many reports in the same time as with one finger, and also use the fingers, like the tips of an open pair of dividers, for measuring magnitudes in the objects. All the same, the circumstance that we have an extended sensitive skin surface, with many sensitive points, recedes wholly into the background when touching things. What we are capable of ascertaining from the skin feeling by gently applying our

<sup>†</sup> [The Latin word means variously 'cast in the way', 'opposed', 'offered' .]

hand, say upon the face of a medal, is extraordinarily rough and scanty in comparison with what we discover by a groping motion, even if only with the point of a pencil. With sight this process becomes much more complicated, because of the fact that besides the most refinedly sensitive spot on the retina – its central fovea – which is as it were led all round the retinal image when we look at something, there also cooperate at the same time a great host of other sensitive points, in a much more fertile manner than is the case with touch.

By moving the touching finger along the objects, one comes to know the sequence in which their impressions offer themselves. This sequence shows itself to be independent of whether one touches with one finger or another. It is moreover not a uniquely determined sequence, whose elements one must always go through, forwards or backwards, in the same order in order to get from one to another; thus it is not a linear sequence, but a surfacelike 'one beside another', or in Riemann's terminology a second-order manifold. That all this is so is easily seen.

Of course, the touching finger can get from one point to another, in the touchable surface, also by other motor impulses than those which push it along the surface; and different touchable surfaces require different movements for sliding upon them. A higher manifold is thereby required for the space in which what touches moves, than for the touchable surface: the third dimension must be added. But this suffices for all available experiences. For a closed surface divides completely the space with which we are acquainted <sup>28</sup>. Even gases and liquids, which after all are not tied to the form of the human faculty of representation <sup>29</sup>, cannot escape through a surface closed all round. And just as only a surface, not a space – thus a spatial structure of two and not three dimensions – can be bounded by a closed line, so also can a surface close off precisely only a space of three dimensions, and not one with four.

Thus might one get to know the spatial order of what exist 'one beside another'. As a further step, magnitudes would be likened with one another, by observing congruence of the touching hand with parts or points of the surfaces of bodies, or congruence of the retina with parts and points of the retinal image.

Because this intuited spatial order of things stems originally from the sequence in which the qualities of sensation offered themselves to the moving sense organ, there finally persists a curious consequence even in

*Summary*

the completed representation of an experienced observer. The objects extant in space namely appear to us clothed in the qualities of our sensations. To us they appear red or green, cold or warm, to have a smell or taste, etc., whereas after all these qualities of sensation belong only to our nervous system and do not reach out at all into external space<sup>30</sup>. The semblance does not cease even when we know this, because in fact this semblance is the original truth: it is indeed sensations which first offer themselves to us in a spatial order<sup>31</sup>.

*The error of the popular belief*

*Transition to geometry*

You can see that the most essential features<sup>32</sup> of spatial intuition can in this way be derived. However, to the consciousness of the general public an intuition appears as something simply given, which comes about without reflection and search, and which is by no means to be resolved further into other psychic processes. This popular belief has been adopted by some workers in physiological optics, and also by the strictly observant Kantians, at least as regards spatial intuition. As is well known, already Kant assumed not only that the general form of spatial intuition is transcendently given, but that it also contains in advance, and prior to any possible experience, certain narrower specifications as expressed in the axioms of geometry<sup>33</sup>. These can be reduced to the following propositions<sup>34</sup>:

(1) Between two points only *one* shortest line is possible. We call such a line “*straight*”.

(2) Through any three points a *plane* can be placed. A plane is a surface which wholly includes any straight line if it coincides with two of its points.

(3) Through any point only one line parallel to a given straight line is possible. Two lines are *parallel* if they are straight lines lying in the same plane which do not intersect within any finite distance.

Indeed, Kant used the alleged fact that these geometrical propositions appeared to us as *necessarily* correct, and that we could never at all even represent to ourselves a deviating behaviour of space, directly as a proof that they had to be given prior to all experience, and that for this reason the spatial intuition contained in them was itself a transcendental<sup>35</sup> form of intuition, independent of experience.

In view of the controversies which have been conducted, in recent years, about the question of whether the axioms of geometry are transcendental or empirical propositions, I should like here to emphasize firstly that this question is wholly to be separated from the one first dis-

cussed, of whether space is in general a transcendental form of intuition or not\*.

Everything our eye sees, it sees as an aggregate of coloured surfaces in the visual field – that is its form of intuition<sup>36</sup>. The particular colours which appear on this or that occasion, their arrangement and sequence – this is the result of external influences and is not determined by any law of our makeup. Similarly from the fact that space is a form of intuiting, nothing whatever follows about the facts expressed by the axioms. If such propositions are taken to be not empirical ones, but to belong instead to the necessary form of intuition, then this is a further particular specification of the general form of space; and those grounds which allowed the conclusion that the form of intuition of space is transcendental, do not necessarily for that reason already suffice to prove, at the same time, that the axioms too are of transcendental origin<sup>37</sup>.

When Kant asserted that spatial relationships contradicting the axioms of Euclid could never in any way be represented, he was influenced by the contemporary states of development of mathematics and the physiology of the senses, just as he was thus influenced in his whole conception of intuition in general as a simple psychic process, incapable of further resolution<sup>38</sup>.

If one wishes to try to represent to oneself something which has never before been seen, one must know how to depict to oneself the series of sense impressions which, according to the known laws of the latter, would have to come about if one observed that object and its gradual alterations successively from every possible viewpoint and with all of one's senses<sup>39</sup>. And at the same time, these impressions must be such that every other interpretation is thereby excluded<sup>40</sup>. If this series of sense impressions can be formulated completely and unambiguously, then one must in my judgement declare that thing to be intuitably representable. Since by presupposition it is a thing which is considered never yet to have been observed, no previous experience can come to our help and guide our fantasy in seeking out the requisite series of impressions; instead, this can only happen by way of the *concept* of the object or relationship to be represented. Such a concept is thus first of all to be elaborated and to be made as specialised as the given purpose requires.

\* See Appendix II below.

*The origin of spatial intuition does not bear on the validity of axioms*

*How to conceive alternative geometries (see the fragment from "On the Origin...")*



The concepts of spatial structures which are taken not to correspond to customary intuition can be reliably developed only by means of calculative analytic geometry. The analytic resources for our present problem were first given by Gauss in 1828 with his essay on the curvature of surfaces, and applied by Riemann in seeking out the logically possible self-consistent systems of geometry. These investigations have not unsuitably been termed *metamathematical*<sup>41</sup>. One should also note that already in 1829 and 1840 Lobatschewsky worked out, in the customary synthetically intuitive manner, a geometry without the axiom of parallels, and one which concurs completely with the corresponding parts of the more recent analytic investigations. Finally, Beltrami has formulated a method for forming images of metamathematical spaces in parts of Euclidean space, by means of which the specification of their manner of appearance in perspective vision is made fairly easy. Lipschitz has demonstrated that the general principles of mechanics can be carried over to such spaces, so that the series of sense impressions which would come about in them can be completely formulated. With this the intuitability of such spaces, in the sense of the definition of this concept given above, has been shown\*.

But here is where disagreement occurs. I demand for the proof of intuitability only that one should be able to formulate, for every manner of observation, specifically and unambiguously the arising sense impressions, by using if necessary a scientific acquaintance with their laws, from which<sup>†</sup> it would ensue, at least for someone acquainted with these laws, that the thing concerned or relationship to be intuited was in fact present<sup>42</sup>. The task of representing to oneself the spatial relationships in metamathematical spaces indeed demands some practice in understanding analytic methods, perspective constructions and optical phenomena.

This is however in disagreement with the older concept of intuition, which only acknowledges something to be given through intuition if its representation enters consciousness at once with the sense impression, and without deliberation and effort. Our attempts to represent mathematical<sup>††</sup> spaces indeed do not have the ease, rapidity and striking self-evidence with which we for example perceive the form of a room which

\* See my lecture on the axioms of geometry.

<sup>†</sup> [Apparently meaning: 'from which sense impressions' .]

<sup>††</sup> [Helmholtz presumably means 'metamathematical' .]

*Intuition need  
not be easy*

we enter for the first time, together with the arrangement and forms of the objects contained in it, the materials of which these consist, and much else as well. Thus if this kind of self-evidence were an originally given and necessary peculiarity of all intuition, we could not up to now assert the intuitability of such spaces.

Yet we are now confronted with a host of cases, on further reflection, which show that assurance and rapidity for the occurrence of specific representations with specific impressions can also be acquired – even when no such connexion is given by nature. One of the most striking examples of this kind is our understanding of our mother tongue. Its words are arbitrarily or accidentally chosen signs – every different language has different ones. Understanding of it is not inherited, since for a German child who was brought up amongst Frenchmen and has never heard German spoken, German is a foreign language. The child becomes acquainted with the meaning of the words and sentences only through examples of their use. In this process one cannot even make understandable to the child – until it understands the language – that the sounds it hears are supposed to be signs having a sense. Lastly, on growing up it understands these words and sentences without deliberation and effort, without knowing when, where and through what examples it learnt them, and it grasps the finest variations of their sense – often ones where attempts at logical definition only limp clumsily behind.

It will not be necessary for me to multiply examples of such processes – they abound richly enough in daily life. This is precisely the basis of art, and most clearly that of poetry and the graphic arts. The highest manner of intuiting, as we find it in an artist's view, is this kind of apprehension of a new type of stationary or mobile appearance of man and nature. When the traces of like kind which are left behind in our memory by often repeated perceptions reinforce one another, it is precisely the law-like which repeats itself most regularly in like manner, while the incidental fluctuation is erased away. For the devoted and attentive observer, there grows up in this way an intuitive image of the typical behaviour of the objects which have interested him, and he knows as little afterwards how it arose as the child can give an account of the examples whereby it became acquainted with the meanings of words. That the artist has beheld something true emerges from the fact that it

*Unconscious  
learning*

seizes us too with a conviction of its truth, when he presents it to us in an example purified from accidental perturbations. He is superior to us, however, in having known how to cull it from everything accidental, and from every confusion arising in the onward rush of the world.

Thus much just to recall how active this psychic process is in our mental life, from the latter's lowest to its highest stages of development. In previous studies I characterised as *unconscious inferences* the connexions between representations which thereby occur – unconscious, inasmuch as their major premiss is formed from a series of experiences, each of which has long disappeared from our memory and also did not necessarily enter our consciousness formulated in words as a sentence, but only in the form of an observation of the senses. The new sense impression entering in present perception forms the minor premiss, to which there is applied the rule imprinted by the earlier observations<sup>43</sup>. More recently I have avoided the name “unconscious inferences”, in order to escape confusion with the – as it seems to me – wholly unclear and unjustified conception thus named by Schopenhauer and his followers. Yet evidently we are dealing here with an elementary process lying at the foundation of everything properly termed thought, even though it still lacks critical sifting and completion of the individual steps, such as occurs in the scientific formation of concepts and inferences<sup>44</sup>.

#### Summary

Thus as concerns firstly the issue of the origin of the axioms of geometry: the fact that the representation of metamathematical spatial relationships is not easy when experience is lacking, cannot be claimed as a ground against their intuitability. Moreover, the latter is completely demonstrable. Kant's proof for the transcendental nature of the axioms of geometry is thus inadequate. On the other hand, investigation of the empirical facts shows that the axioms of geometry, taken in the only sense in which one is allowed to apply them to the actual world, can be empirically tested and demonstrated, or even – if the case should arise – refuted\*.

The memory vestiges of previous experiences also play a further and highly influential role in the observation of our visual field. A no longer completely inexperienced observer receives even without movement of his eyes – whether by momentary illumination from an electric discharge

\* See my *Wissenschaftliche Abhandlungen*, vol. II, p. 640; an excerpt is given as Appendix III below.

or by deliberate rigid staring – a relatively rich image of the objects in front of him. Yet even an adult will still easily convince himself that this image becomes much richer, and especially much more precise, when he moves his glance around in the visual field, and thus employs that kind of spatial observation which I described earlier as the fundamental one. We are indeed also so used to letting our glance wander upon the objects we observe, that it requires a fair amount of practice before we succeed, for the purposes of experiments in physiological optics, in holding it fixed upon one point for a longish time without wavering.

In my works on physiological optics\*, I have sought to explain how our acquaintance<sup>†</sup> with the visual field can be acquired by observation of the images during the movements of our eyes, provided only that there exists, between otherwise qualitatively alike retinal sensations, some or other perceptible difference corresponding to the difference between distinct places on the retina. Such a difference should be called a *local sign*, according to Lotze's terminology<sup>45</sup>; except that the fact that this sign is a local sign – i.e. corresponds to a difference of place and to which such difference – need not be known in advance.

Recent observations\*\* have also reconfirmed that persons who were blind from youth onwards, and later regained their sight through an operation, could not at first distinguish by eye even between such simple forms as a circle and a square, until they had touched them.

Apart from this, physiological investigation teaches us that we can liken by visual estimation, in a relatively precise and assured manner, only such lines and angles in the visual field as can be brought, by normal eye motions, to form images in rapid succession at the same places on the retina. Indeed, we estimate much more assuredly the true magnitudes and distances of spatial objects situated not too far off, than the perspective ones, alternating according to viewpoint, in the visual field of the observer – although the former task concerns the three dimensions of space and is much more involved than the latter, which concerns only a surfacelike

*"Empirism"  
about visual  
perception*

\* *Handbuch der Physiologischen Optik* [‘Handbook of physiological optics’]; *Vorträge über das Sehen der Menschen* [‘Lectures on human sight’], in *Vorträge und Reden*, vol. I, pp. 85 and 265.

<sup>†</sup> [‘Kenntnis’ : note the end of the first paper in this collection, where Helmholtz contrasts knowledge (‘Erkenntnis’) of a conceptual connexion and an intuitive acquaintance deriving from frequent observation.]

\*\* Dufour (Lausanne) in the *Bulletin de la Société médicale de la Suisse Romande*, 1876.

image. One of the greatest difficulties in drawing, as is well known, is to free oneself from the influence involuntarily exerted by our representation of the true magnitudes of the objects seen. Now it is precisely the situation described that we must expect, if our understanding of local signs was first acquired through experience. We can assuredly become acquainted with the alternating sensory signs for what remains objectively constant, much more easily than with those for what alternates according to every single movement of our body, as the perspective images do.

#### Nativism

There is, none the less, a large number of physiologists whose view we may term *nativist*, as opposed to the *empiricist* view which I myself have tried to defend, and for whom this conception of an acquired acquaintance with the visual field appears unacceptable. This is due to their not having got clear about what after all lies before us so plainly in the example of speech, namely how much can be achieved by accumulated memory impressions. For this reason, a host of various attempts have been made to reduce at least some part of visual perception to an innate mechanism, in the sense that certain impressions of sensation are supposed to release certain ready-made spatial representations.

I have demonstrated in detail\* that all hypotheses of this kind proposed to date are inadequate, because in the end one can still always come up with cases where our visual perception is in more precise agreement with actuality than those assumptions would yield. One is then forced to add the further hypothesis that the experience gained during movements can in the end overcome the innate intuition, and thus achieve *in opposition* to the latter what it is supposed by the empiricist hypothesis to achieve *without* such an obstacle.

#### Rejection of nativism

The nativist hypotheses about our acquaintance with the visual field thus *firstly* do not explain anything, but simply assume the existence of the fact to be explained while at the same time rejecting the possibility of reducing this fact to definitely ascertained psychic processes, although they themselves still have to appeal to the latter in other cases. *Secondly*, the assumption of every nativist theory – that ready-made representations of objects are elicited through our organic mechanism – appears much more audacious and doubtful than the assumption of the empiricist

theory, which is that only the non-understood material of sensations originates from external influences, while all representations are formed from it in accordance with the laws of thought.

*Thirdly*, the nativist assumptions are unnecessary. The only objection that one has been able to bring against the empiricist explanation, is the assurance with which many animals move when newly born or just after crawling out of the egg<sup>46</sup>. The less mentally endowed they are, the quicker they learn what they at all can learn. The narrower the paths are along which their thoughts must go, the more easily they find them. A newly born human child is extremely inept at seeing: it needs several days before learning to judge, from its visual images, in what direction it must turn its head in order to reach its mother's breast. Young animals are certainly much more independent of individual experience. But we still know practically nothing specific about what this instinct which guides them is: whether direct inheritance is possible of ranges of representations from the parents, or whether it is only a matter of desire or aversion – or of a motor impulse – which attach themselves to certain sensation aggregates. Vestiges of the last mentioned phenomena still occur in a plainly recognisable manner with human beings. Properly and critically executed observations would be in the highest degree desirable in this domain.

Thus for the kind of set-up presupposed by the nativist hypothesis, one can at most claim a certain pedagogical merit which facilitates detection of the first lawlike relationships. The empiricist view too could be combined with presuppositions having this aim, for example that the local signs of neighbouring places on the retina are more similar to each other than are those of ones further apart, that those of corresponding places on the two retinas are more similar than those of disparate ones, and so on. For our present investigation it suffices to know that spatial intuition can come fully into being even with a blind person, and that with a sighted person – even should the nativist hypotheses prove partially correct – the final and most precise specification of spatial relationships is still conditioned by the observations made during movement<sup>47</sup>.

I shall now return to our discussion of the initial, original facts of our perception. As has been seen, we do not merely have alternating sense impressions which come upon us without our doing anything about it. We rather observe during our own continuing activity, and thereby attain

*There is a causal link between our immersions and spatial order*

\* See my *Handbuch der Physiologischen Optik* [op. cit.], 3. Abteilung, Leipzig, 1867.

an acquaintance with the *enduring existence* of a lawlike relationship between our innervations and the becoming present of the various impressions from the current range of presentables. Each of our voluntary movements, whereby we modify the manner of appearance of the objects, is to be regarded as an experiment through which we test whether we have correctly apprehended the lawlike behaviour of the appearance before us, i.e. correctly apprehended the latter's<sup>†</sup> presupposed enduring existence in a specific spatial arrangement.

The chief reason, however, why the power of any experiment to convince is so much greater than that of observing a process going on without our assistance, is that with the experiment the chain of causes runs through our own self-consciousness. We are acquainted with one member of [the chain of] these causes – the impulse of our will – from inner intuition, and know through what motives it came about<sup>48</sup>. From this, as from an initial member known to us and at a point in time known to us, there then begins to act that chain of physical causes which terminates in the outcome of the experiment. Yet the conviction to be attained has an essential presupposition, that the impulse of our will should neither itself already have been influenced by physical causes which at the same time also determined the physical process, nor should it for its own part have influenced the subsequent perceptions psychically.

*Cannot spatial  
order be  
generated by us?*

The latter doubt can in particular be of relevance to our present topic. The impulse of the will for a specific movement is a psychic act, just as is the thereupon perceived alteration in the sensation. Could not then the first act bring about the second through purely psychic mediation?

It is not impossible. When we dream, something of the sort occurs. In our dream we believe ourselves to execute a movement, and we then dream further that there occurs what should be its natural consequence. We dream of climbing into a boat, pushing it off from land, gliding out on the water, seeing the displacement of the surrounding objects, and so on. Here the dreamer's expectation that he will see the consequences of his conduct occur appears to bring about the dreamed perception in a purely psychic way. Who can say how long and finely spun out, how logically complete such a dream might be? Should everything in it occur in the most lawlike manner according to the order of nature, there would

<sup>†</sup> [or possibly 'their', referring to 'the objects' rather than 'the appearance'.]

remain but one difference from the waking state – the possibility of being awakened, the rupture of this dreamed series of intuitions.

I do not see how we could refute a system of even the most extreme subjective idealism, if it regarded life as a dream<sup>49</sup>. We might declare it to be as improbable and unsatisfying as possible – in this respect I would assent to the sharpest expressions of repudiation. But it could be implemented consistently, and it seems to me very important to keep this in view. It is well known how ingeniously Calderón implemented this theme in his 'Life a Dream'.

*No refutation  
of idealism*

Fichte too assumes that the 'I' posits the 'not-I' – i.e. the world as it appears – for itself, because it needs it for developing its own thought-activity. Yet his idealism does indeed distinguish itself from that just referred to, in that he conceives of other human individuals not as dream images, but – starting from the assertion of the moral law – as essences alike with one's own 'I'<sup>50</sup>. Since, however, the images whereby they each represent the 'non-I' must themselves all agree with one another, he conceives of all of the individual 'I's' as parts or emanations of the absolute 'I'. The world in which they found themselves was then that world of representations which the worldmind posited for itself, and could again receive the concept of reality, as happened with Hegel.

The *realist* hypothesis, on the other hand, trusts the testimony of ordinary self-observation, according to which the alterations of perception which follow some item of conduct have no psychic connexion at all with the preceding impulse of the will. It regards as enduring, independent of the way in which we form representations<sup>†</sup>, that which seems to prove to be thus in everyday perception – the material world outside us.

*Reasons to  
prefer realism*

The realist hypothesis is the simplest we can form, it has been tested and confirmed in extraordinarily wide ranges of application, it is sharply defined for every individual specification, and it is therefore extraordinarily serviceable and fruitful as a basis for conduct. All this is without doubt. Even in the idealist manner of conceiving things, we would hardly know how else to express the lawlike in our sensations than by saying: "Those acts of consciousness which occur with the character of perception proceed *as if* there actually existed the world of material things which is assumed by the realist hypothesis."<sup>51</sup> But we cannot get beyond this 'as if'. We cannot acknowledge the realist view to be more than an excellently serviceable and precise hypothesis. We are not allowed to

*No conclusive  
proof*

<sup>†</sup> ['von unserem Vorstellen'.]



ascribe to it necessary truth, since besides it idealist hypotheses not open to refutation are also possible.

It is good to keep this always before our eyes, so that we may not wish to infer more from the facts than there is to infer from them. The various gradations of the idealist and realist views are metaphysical hypotheses. As long as they are acknowledged to be such, they are ones which have their complete scientific justification, however harmful they may become when one wishes to present them as dogmas or alleged necessities of thought.

*Criteria of scientific choice* Science must discuss all admissible hypotheses, in order to retain a fully comprehensive view of the possible attempts at explanation. Hypotheses are even more necessary for conduct, because one cannot continually wait until an assured scientific decision has been reached, but must decide for oneself – whether according to probability or to aesthetic or moral feeling. In this sense one could have no objection even against metaphysical hypotheses. But it is unworthy of a thinker wishing to be scientific if he forgets the hypothetical origin of his propositions. When such concealed hypotheses are defended with pride and passionateness, the latter are the customary consequences of the unsatisfying feeling which their defender shelters, in the hidden depths of his conscience, about the justness of his cause.

*We can directly perceive lawlike connections* Yet what we can find unambiguously, and as a fact without anything being insinuated hypothetically, is the lawlike in the phenomena. From the first step onwards, when we perceive before us the objects distributed in space, this perception is the acknowledgement of a lawlike connexion between our movements and the therewith occurring sensations. Thus even the first elementary representations contain intrinsically some thinking, and proceed according to the laws of thought. Everything in intuition which is an addition to the raw material of sensations can be resolved into thinking, if we take the concept of thinking as broadly as has been done above<sup>52</sup>.

For if ‘comprehending’ means forming *concepts*<sup>†</sup>, and in the concept of a class of objects we gather together and bind together whatever like characteristics they bear, it then results quite analogously, that the concept of a series of appearances alternating in time must seek to bind to-

<sup>†</sup> [Here there is a play on the words ‘begreifen’ and ‘Begriff’ which is untranslatable, as the former means more than merely ‘conceiving’.]

gether that which remains alike in all of its stages<sup>53</sup>. The wise man, as Schiller puts it:

Seeks the familiar law  
in chance’s frightful miracles,  
Seeks the stationary pole  
in the fleeting appearances.<sup>†</sup>

That which remains alike, without dependence upon anything else, through every alternation of time, we call *substance*. The relationship which remains alike between altering magnitudes, we call the *law* connecting them. What we perceive directly is only this law<sup>54</sup>. The concept of substance can be gained only through exhaustive examination and always remains problematic, inasmuch as further examination is not ruled out. Formerly light and heat were counted as substances, until it later turned out that they are perishable forms of motion. And we must still always be prepared for new decompositions of the currently familiar chemical elements.

The first product of the thoughtful comprehension of the phenomena is lawlikeness. Should we have separated it out sufficiently purely, delimited its conditions with sufficient completeness and assurance and also formulated them with sufficient generality that the outcome is unambiguously specified for all possibly occurring cases, and that we at the same time gain the conviction that it has proved true and will prove true at all times and in every case: then we acknowledge it as an existence enduring independent of the way in which we form representations, and call it the *cause*, i.e. that which primarily remains and endures behind what alternates. In my opinion, only the application of the word in this sense is justified, although it is applied in common speech in a very wishy-washy manner for whatever at all is the antecedent or occasion of something<sup>55</sup>.

*Causation is a form of lawlikeness*

Inasmuch as we then acknowledge the law to be something compelling our perception and the course of natural processes, to be a power equivalent to our will, we call it ‘force’<sup>56</sup>. This concept of a power con-

<sup>†</sup> Sucht das vertraute Gesetz  
in des Zufalls grausenden Wundern,  
Sucht den ruhenden Pol  
in der Erscheinungen Flucht.

[The contexts show that Helmholtz intends this and subsequent snatches of poetry in their most literal sense. So the relevant literal sense is given in this translation.]

fronting us is conditioned directly by the way in which our simplest perceptions come about. From the beginning, those changes which we make ourselves by acts of our will are distinct from ones which cannot be made by our will, and are not to be set aside by our will. Pain especially gives us the most penetrating lesson about the power of reality. Emphasis thereby falls upon the observational fact that the perceived range of presentables is not posited by a *conscious* act of our representation or will. Fichte's 'not-I' is here the exactly fitting negative expression<sup>57</sup>. For the dreamer too, what he believes himself to see and feel appears not to be evoked by his will or by conscious concatenation of his representations, although the latter might in actuality often enough be unconsciously the case. For him too it is a 'not-I'. Likewise for the idealist, who regards it as the world-mind's world of representations.

In our language, we have a very fortunate way of characterising that which lies behind the change of appearances and acts upon us, namely as 'the actual'. Here only action<sup>†</sup> is predicated. Absent is that secondary reference to what endures as substance which is included in the concept of the real, i.e. of the thinglike. As regards the concept of the objective, on the other hand, the concept of a ready-made image of an object usually finds its way into it, and one which does not suit the most primary perceptions.

Even with the logical dreamer, we must presumably characterise as active and actual those psychic states or motives which foist upon him, at the given time, the sensations corresponding in a lawlike manner to the present situation in his dream world. It is clear, on the other hand, that a division between what is thought and what is actual does not become possible until we know how to make the division between what the 'I' can and cannot alter. This does not become possible, however, until we discern what lawlike consequences the impulses of our will have at the given time. The lawlike is therefore the essential presupposition for the character of the actual.

I need not explain to you that it is a *contradictio in adjecto* to want to represent the real, or Kant's 'thing in itself', in positive terms but

<sup>†</sup> [Here (and as far as possible elsewhere) the following equivalences are used: wirklich = actual, Wirken = action, (ein)wirken = to act, wirksam = active, reell = real, sachlich = thinglike, objektiv = objective, Gegenstand = object. Elsewhere 'Wirkung' is generally translated by 'effect' and 'Einwirkung' by 'influence'.]

without absorbing it into the form of our manner of representation. This is often discussed. What we can attain, however, is an acquaintance with the lawlike order in the realm of the actual, admittedly only as portrayed in the sign system of our sense impressions:

Everything perishable  
Is only a likeness.<sup>†</sup>

Goethe, *Faust*

I take it as a favourable sign that we find Goethe, here and further on, together with us on the same path. Where it is a matter of broad panoramas, we may well trust his clear and unconstrained eye for truth. He demanded from science that it should be only an artistic arrangement of the facts and form no abstract concepts going beyond this, which to him seemed to be empty names and only obscured the facts. In somewhat the same sense, Gustav Kirchhoff has recently characterised it as the task of the most abstract amongst the natural sciences, namely mechanics, to *describe completely and in the simplest manner* the motions occurring in nature<sup>58</sup>.

As for 'obscuring', this indeed happens when we stay put in the realm of abstract concepts and do not explain to ourselves their factual sense, i.e. make clear to ourselves what observable new lawlike relationships between the appearances follow from them. Every correctly formed hypothesis sets forth, as regards its factual sense, a more general law of the appearances than we have until now directly observed – it is an attempt to ascend to something more and more generally and inclusively lawlike. Whatever factually new things it asserts must be tested and confirmed by observation and experiment. Hypotheses not having such a factual sense, or which in no way specify anything sure and unambiguous about the facts falling under them, are to be regarded only as worthless talk.

Every reduction of the appearances to the underlying substances and forces claims to have found something unchanging and final. An unconditional claim of this kind is something for which we never have a justification: this is allowed neither by the fact that our knowledge is full of gaps, nor by the nature of the inductive inferences upon which all of our perception of the actual, from the first step onwards, is based.

<sup>†</sup> Alles Vergängliche  
Ist nur ein Gleichnis.

*We cannot  
perceive reality,  
but we can  
perceive its causal  
order*

*Science should be  
restricted to  
dealing with  
causal phenomena*

*Progress in our  
knowledge of  
causes*

*No knowledge of  
reality*

*Reality  
(actuality) is  
characterised by  
being beyond our  
power*

*Causality as a  
condition of  
understanding*

Every inductive inference is based on trusting that an item of lawlike behaviour, which has been observed up to now, will also prove true in all cases which have not yet come under observation. This is a trust in the lawlikeness of everything that happens. However, lawlikeness is the condition of comprehensibility. Trust in lawlikeness is thus at the same time trust in the comprehensibility of the appearances of nature. While: should we presuppose that this comprehension will come to completion, that we shall be able to set forth something ultimate and finally unalterable as *the cause* of the observed alterations, then we call the regulative principle of our thought which impels us to this the *law of causality*<sup>†</sup>. We can say that it expresses a trust in the *complete comprehensibility* of the world.

*Practical  
evidence of our  
understanding  
of the world*

Comprehension, in the sense in which I have described it, is the method whereby our thought masters the world, orders the facts and determines the future in advance. It is its right and its duty to extend the application of this method to everything that occurs, and it has already actually harvested great yields on this path. However, we have no further guarantee for the applicability of the law of causality than this law's success. We could live in a world in which every atom was different from every other one, and where there was nothing at rest. Then there would be no regularity of any kind to be found, and our thought activity would have to be at a standstill<sup>59</sup>.

*See Schlick's  
notes 59 and 60*

The law of causality actually is an *a priori* given, a transcendental law<sup>60</sup>. A proof of it from experience is not possible, since the first steps of experience, as we have seen, are not possible without employing inductive inferences, i.e. without the law of causality. But even suppose that complete experience could tell us – though we are still far from being entitled to affirm this – that everything so far observed had occurred in a lawlike manner. It would still follow from such experience only by an inductive inference, i.e. by presupposing the law of causality, that the law of causality would then also hold in the future. Here the only valid advice is: have trust and act!

The inadequate  
It then takes place.<sup>††</sup>

That should perhaps be the answer given by us to the question: what

<sup>†</sup> [ 'Kausalgesetz' ]

<sup>††</sup> Das unzulängliche  
Dann wird's Ereignis.

*Goethe, Faust*

is true in the way in which we form representations? Regarding what has always seemed to me to be the most essential advance in Kant's philosophy, we still stand on the ground of his system. In this regard I have also frequently emphasized in my previous studies the agreement<sup>61</sup> between recent physiology of the senses and Kant's doctrines, although this admittedly does not mean that I had to swear by the master's words in all subordinate matters too. I believe the resolution of the concept of intuition into the elementary processes of thought as the most essential advance in the recent period. This resolution is still absent in Kant, which is something that then also conditions his conception of the axioms of geometry as transcendental propositions. Here it was especially the physiological investigations on sense perceptions which led us to the ultimate elementary processes of cognition. These processes had to remain still unformulable in words, and unknown and inaccessible to philosophy, as long as the latter investigated only cognitions finding their expression in language.

Admittedly – for those philosophers who have retained the inclination for metaphysical speculations, the most essential thing in Kant's philosophy appears to be precisely what we have considered to be a defect hanging upon inadequate development in the specialised sciences of his time. Kant's proof, indeed, for the possibility of a metaphysics – and of course he himself did not know how to discover anything more about this alleged science – relies purely and simply on the belief that the axioms of geometry, and the related principles of mechanics, are transcendental propositions given *a priori*. Moreover his whole system properly speaking contradicts the existence of metaphysics, and the obscure points of his epistemology, about whose interpretation there has been so much controversy, derive from this root.

According to all this, science would seem to have its own secure territory standing firmly upon which it can seek for the laws of the actual – a marvellously rich and fruitful field of work. As long as it restricts itself to this activity, it will be unaffected by idealist doubts. Such work may appear modest in comparison with the soaring schemes of metaphysicians.

Yet with gods  
Shall measure himself

*The secure  
foundation of  
science in  
exploring the  
causal order*

*Somewhere in  
Goethe*

No mortal.  
 If he raises himself up  
 And touches the stars  
 With the crown of his head,  
 Then nowhere cling  
 His uncertain soles,  
 And there play with him  
 Clouds and winds.

If he stands with firm  
 Pithy bones  
 On the well-founded  
 Lasting Earth:  
 In height he does not reach  
 Even with the oak  
 Or the vine  
 To liken himself.<sup>†</sup>

All the same, the example of the man who said this may teach us how a mortal, who had surely learnt how to stand, even when he touched the stars with the crown of his head, still retained a clear eye for truth and actuality. Something of the artist's vision, of the vision which led Goethe and also Leonardo da Vinci to great scientific thoughts, is what the genuine enquirer must always have. Both artist and enquirer strive, although with different approaches, towards the goal of discovering new lawlikeness. Only, one must not try to pass off idle enthusiasm and crazy fantasies as an artist's vision. The genuine artist and the genuine enquirer both know how to work genuinely, and to give their works a firm form and a convincing fidelity to truth.

<sup>†</sup> Doch mit Göttern  
 Soll sich nicht messen  
 Irgendein Mensch.  
 Hebt er sich aufwärts  
 Und berührt  
 Mit dem Scheitel die Sterne,  
 Nirgends haften dann  
 Die unsicheren Sohlen,  
 Und mit ihm spielen  
 Wolken und Winde.

Steht er mit festen  
 Markigen Knochen  
 Auf der wohlgegründeten  
 Deuernden Erde:  
 Reicht er nicht auf,  
 Nur mit der Eiche  
 Oder der Rebe  
 Sich zu vergleichen.

Moreover, actuality has so far always revealed itself much more sublimely and richly to a science enquiring in a manner faithful to its laws, than the utmost efforts of mythical fantasy and metaphysical speculation had known how to depict it. What have all the monstrous offspring of Indian reverie, the piling up of gigantic dimensions and numbers, to say as against the actuality of the structure of the universe, as against the intervals of time in which sun and earth were formed, in which life evolved during geological history and adapted itself, in more and more perfect forms, to the more stable physical situations on our planet?

What metaphysics has prepared, in advance, concepts of effects such as magnets and moving electricity exert on each other? Physics at this moment is still striving to reduce them to well-specified elementary effects, without having reached a clear conclusion. But already light too seems to be nothing other than one more kind of motion of these two agencies, and the aether filling space is acquiring wholly new characteristic properties as a magnetisable and electrifiable medium.

And into what scheme of scholastic concepts shall we insert that supply of effective energy whose constancy the law of the conservation of force asserts? This supply, like a substance, cannot be destroyed or increased; it is at work as a driving force in every motion of both lifeless and living matter; it is a Proteus ever attiring itself in new forms, active throughout infinite space and yet not divisible by space without remainder, the effective factor in every effect, the motive factor in every motion – and yet not mind and not matter. Did the poet have a presentiment of it?

In life's currents, in a storm of deeds,  
 I float up and down,  
 Weave here and there!  
 Birth and grave,  
 An eternal ocean,  
 An alternating weaving,  
 A glowing life,  
 Thus I create on the humming loom of time  
 And make the deity's living garment.<sup>†</sup>

*Goethe, Faust*

<sup>†</sup> In Lebensfluten, in Tatensturm, Wall' ich auf und ab, Webe hin und her! Geburt und Grab,	Ein ewiges Meer, Ein wechselnd Weben, Ein glühend Leben, So schaff' ich am sausenden Webstuhl der Zeit, Und wirke der Gottheit lebendiges Kleid.
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We are particles of dust on the surface of our planet, which itself is barely to be called a grain of sand in the infinite space of the universe. We are the youngest generation of living things on earth, by the geological reckoning of time barely arisen from the cradle, still at the stage of learning, barely half-educated, and pronounced of age only out of mutual considerateness. Yet we have already – through the more powerful impulse of the law of causality – grown out above all of our fellow-creatures and are subduing them in the struggle for existence. We truly have sufficient ground to be proud that it has been given to us slowly to learn to understand, by faithful labour, “the incomprehensibly high works”. And we need not feel in the least ashamed of not succeeding in this immediately at the first assault of a flight like that of Icarus.

## APPENDICES

1. ON THE LOCALISATION OF THE SENSATIONS  
OF INTERNAL ORGANS

The issue might arise here of whether the physiological and pathological sensations of internal organs of the body should not fall into the same category as psychic states, inasmuch as many of them are likewise not altered by movements, or at least not altered considerably<sup>62</sup>.

Now there are indeed sensations of an ambiguous character, such as those of depression, melancholy and anxiety, which may just as well arise from bodily causes as from psychic ones, and for which there is also lacking any representation of a particular localisation<sup>63</sup>. At most, in the case of anxiety, the region of the heart vaguely asserts a claim to be the seat of the sensation, as in general the older view making the heart the seat of many psychic feelings was obviously derived from the fact that the movement of this organ is often altered by such feelings, a movement which one feels partly directly and partly indirectly through superimposing one's hand. So there thus arises a kind of false bodily localisation for what are actually psychic states. In states of illness this goes much further. I recall having seen, as a young doctor, a melancholic shoemaker

But the axioms of geometry limit the form of intuition of space in such a way that it can no longer absorb every thinkable content, if geometry is at all supposed to be applicable to the actual world. If we drop them, the doctrine of the transcendental of the form of intuition of space is without any taint<sup>87</sup>. Here Kant was not critical enough in his critique; but this is admittedly a matter of theses coming from mathematics, and this bit of critical work had to be dealt with by the mathematicians.

## NOTES AND COMMENTS

<sup>1</sup> The rector's address on 'The Facts in Perception', given on the occasion of the anniversary celebrations of Berlin University in 1878, rightly counts as the richest in content amongst Helmholtz' epistemological studies. Although Koenigsberger's comment (*H. v. Helmholtz*, vol. II, p. 78) that Helmholtz here gave "a self-contained system of his philosophy" seems to go a little too far, it is certain enough that the lecture contains the most complete and rounded off presentation of his epistemology. Form and content are on a level such that one can willingly concur with Koenigsberger's declaring the study to be the "finest and most important" of his addresses (*ibid.*, p. 246).

How great a fundamental importance Helmholtz himself ascribed to the subject of the address is perhaps best discernible from the sonorous titles which he passingly considered for introducing the lecture. "I shall devise the title last of all," he writes to his wife, "I do not yet know it. Perhaps 'What actually exists?' or 'Everything Perishable is only a Likeness' or 'A Journey to the Mothers'." (*ibid.*, p. 246)

<sup>2</sup> The predicate 'true' should be ascribed in the strictest sense only to judgements (statements). One can thus ask about the truth of *thought*, to the extent that 'thinking' is intended to mean the same as 'judging'. But if we talk about what is true in our *intuition*, this manner of speech refers only to judgements *about* our intuition, or to judgements – should there be such – which intuition contains in a concealed manner.

<sup>3</sup> This question can be included under the preceding one, since the importance for scientific enquiry and for life of the relationship between our intuitions and actuality is precisely that one must be acquainted with this relationship in order to pass true judgements upon actuality.

<sup>4</sup> Helmholtz' conceptions seem to presuppose that one somehow defines philosophy as the science of the *mental*. Yet such a definition would hardly be suitable, since the mental – here obviously thought identical with the 'psychic' – indeed forms the object of a particular science, namely psychology. Of course, a simple identification of psychological and philosophical enquiry has occasionally been attempted (thus Theodor Lipps wanted to define philosophy as "mental science" or the science of inner experience"). But in this way

<sup>1</sup> [ 'Geisteswissenschaft' : the distinction at English-speaking universities between 'arts' and 'sciences' is spoken of in German as a distinction between 'mental sciences' and 'natural sciences'. Lipps obviously has the literal sense of this term in mind, and M.S. alludes to it in his comments.]

one assuredly neither does justice to philosophy nor to psychology: many of course, and not without justification, consider psychology to be a branch of natural science. Without here going into the controversial and basically unessential issue of the definition of philosophy, we can say that the wholly correct thought that Helmholtz here has in mind would be better formulated as follows: the natural sciences and the mental sciences, when they pursue their proper issues down to ultimate principles, necessarily end up in epistemology and meet there, thus in the domain of philosophy.

One also has to reach this domain to decide whether, or in what sense, the customary distinction between the mental and the corporeal can be maintained. When Helmholtz contrasts here the “influences of the corporeal world” and the “mind’s own workings”, he thereby attaches himself verbally to the popular conception of the world. Whether or not he thereby perhaps also uncritically introduces presuppositions having a bearing on the matter, will only emerge in the further course of the discussion (compare below note 49).

<sup>5</sup> One must be carefully on one’s guard against misunderstanding the meaning of the term ‘prior to experience’ in Kant’s philosophy. The word ‘prior’ has namely two meanings, since it may be understood temporally or logically. For Kant, *a priori* knowledge is only in the *logical* sense *prior* to experience, i.e. its validity is not dependent upon experiences, does not have its logical ground in them. Kant says emphatically (*Kritik der reinen Vernunft* [‘Critique of pure reason’], 2nd ed., p. 1): “Thus in respect of time no knowledge comes about in us prior to experience, and any such begins with the latter. But though all of our knowledge starts *with* experience, it yet does not therefore simply all arise *out of* experience.”<sup>†</sup>

<sup>6</sup> In place of ‘transcendental’ one should properly have ‘*a priori*’. It has already been mentioned (note 1.55) that Helmholtz often exchanged the two terms. For Kant *a priori* is whatever is valid independent of experience. The word transcendental has a somewhat more complicated meaning. Kant’s own explanation of it (*Kritik der reinen Vernunft* [op. cit.], 2nd ed., p. 25) runs: “All knowledge which deals not so much with objects, as rather with our manner of knowing objects, insofar as this is taken to be one possible *a priori*, I call transcendental.” And at another point (*ibid.*, p. 81) he says expressly: “Therefore neither space, nor any geometrical specification of it *a priori*, is a transcendental representation; but rather only the knowledge that these representations are not at all of empirical origin, and the possibility whereby it can<sup>††</sup> nonetheless relate *a priori* to objects of experience, can be called transcendental.”

<sup>7</sup> According to Locke (1632–1704) there are two<sup>†††</sup> kinds of properties. Namely the quantitatively conceivable spatial and temporal ones, which in his view belong to things themselves and are termed by him ‘primary’ properties, and the ‘secondary qualities’ such as red, cold, loud and so on. These differ from the former in being only modifications of our perceiving consciousness, in which they are produced by the influence of bodies upon our sense organs. They are thus not properties of things at all, but belong only to the subject and have no similarity to the true properties of external things. This is the doctrine of the ‘subjectivity of the qualities of the senses’, which was already quite clearly expressed

<sup>†</sup> [Kemp Smith’s translation is not followed exactly.]

<sup>††</sup> [Some read ‘können’ for ‘könne’, i.e. ‘they can’ instead of ‘it can’.]

<sup>†††</sup> [In fact three kinds: see remark appended to note 24 below.]

in antiquity by Democritus. In recent times it was already again advocated before Locke, and probably first of all by Galileo.

<sup>8</sup> What Johannes Müller termed the law of specific sensory energies was admittedly not formulated by him in a wholly unobjectionable manner. One can perhaps express it more carefully thus: the modality of the sensation depends in an immediate manner only upon what region of the central organ is put into a corresponding excited state, independent of the external causes bringing about the excitation. Probably still better and more general formulations are possible – but what is properly the basic thought, one expressing nothing other than the doctrine of the subjectivity of the qualities of the senses in a physiological formulation, remains the same and remains correct. The attacks *in principle* which have been levelled against J. Müller’s law, e.g. by W. Wundt in his psychology, are therefore in any case unjustified.

<sup>9</sup> One no longer today, according to the electromagnetic theory of light (to whose victory precisely Helmholtz himself contributed), conceives of light oscillations as material motions, and they are consequently regarded as essentially different from sound waves. But this fact is of no significance for the comparison drawn by Helmholtz here.

<sup>10</sup> According to the theory of Young and Helmholtz, there are three different basic *processes* in the retina of the eye, which correspond to the sensations red, green and blue. The sensations of the remaining colours occur, according to this theory, when more than one of these processes are excited at once. But psychologically speaking, every colour sensation is doubtless something simple, unanalysable. One should thus strictly only talk of a mixture of the physiological processes, not of a mixture of the sensations or ‘in sensation’. We shall find on a number of further occasions that Helmholtz did not distinguish strictly enough between the physiological process of sensory excitation and the psychological process of sensation.

It may be noted incidentally that modern theories of colour (Hering, v. Kries, G. E. Müller), in order to achieve a better fit with the empirical facts, have deviated not inconsiderably from the view of Young and Helmholtz. Yet that is wholly inessential as regards the epistemological connexions.

<sup>11</sup> Helmholtz’ theory of consonance [harmony], to which he refers in this paragraph, has not remained unchallenged. Stumpf in particular, in his *Tonpsychologie* [‘Psychology of sound’], has developed a view based on essentially different foundations. But once again: in this connexion the correctness of the psycho-physiological theory is not involved at all, as it only has to fulfil the task of an explanatory example.

<sup>12</sup> The formulation of this sentence could be attacked from various aspects. Here we shall only point out that terming sensations effects ‘in our organs’ is dubious, since – as psychic magnitudes – they are obviously not in our organs so much as in our *consciousness*. We have before us the substitution of one thing for another which was already touched upon a moment ago in note 10, and one which would remain harmless only if one could demonstrate that Helmholtz always understood the word sensation<sup>†</sup> only ‘in a physical respect’,

<sup>†</sup> [‘Empfindung’: this word has no etymological connexion with the German equivalents of ‘sense’ (noun) and ‘sense organ’, although it generally corresponds to ‘sensation’ and the verb ‘empfinden’ to ‘to sense’.]

as a process in the sense organ. (Compare B. Erdmann, *Die philosophischen Grundlagen von Helmholtz' Wahrnehmungstheorie* ['The philosophical foundations of Helmholtz' theory of perception'], p. 19.) Where he does this, he contrasts with it the corresponding psychic datum as a 'perception'. It is put thus in an early note made known by Koenigsberger (*Hermann von Helmholtz*, vol. II, p. 129): "Perception is the becoming conscious of a certain sensation, i.e. of a certain state of our organs." Yet he seems all the same not to adhere consistently to this terminology and manner of thought.

<sup>13</sup> It is not only the expression of the sensation, but the latter itself which is different; thus instead of "how such an effect expresses itself" one should indeed rather say "what sort of effect occurs".

<sup>14</sup> In accordance with note 2 above, talk about truth of *images* is to be understood as an abbreviated manner of speech whose sense is: truth of judgements which assert likeness of objects with images. Elsewhere (1st ed. of the *Physiologische Optik* [op. cit.], reprinted 3rd ed., vol. III, p. 18) Helmholtz analogously explains true representations to be such as lead us to suitable<sup>†</sup> conduct; successful conduct is namely an indication of the correctness of the judgements which are taken as a basis: "It is therefore my opinion, that there can be no possible sense at all in speaking of any other kind of truth of our representations than a *practical* one. Our representations of things *cannot* be anything at all other than symbols – naturally given signs for the things which we learn to use for regulating our movements and conduct. If we have learnt to read those symbols correctly, we are in a position to arrange our conduct with their help such that it will have the desired outcome, i.e. such that the expected new sensations occur. Not only is there *in actuality* no other manner of likening representations and things, as all schools are agreed, but none other is even *thinkable* at all and has at all any sense."

<sup>15</sup> In Schlick, *Allgemeine Erkenntnislehre* ['General epistemology'], part I, an attempt is made to show that forming such an image of what is lawlike in the actual, with the help of a sign system, altogether constitutes the essence of all knowledge, and that therefore our cognitive process<sup>††</sup> can only in this way fulfil its task and needs no other method for doing so.

<sup>16</sup> The way in which this sentence is formulated, together with what follows, creates the impression that Kant declared the qualities of sensation to be a 'form of intuition'. That is not the case by any means. Only space and time are forms according to Kant, while the qualities of sensation are for him always *contents* of intuition, and have a wholly different significance for cognition from what the forms have (only the latter are namely for him sources of synthetic judgements *a priori*). Helmholtz' remark is simply intended to signify that the qualities of sensation are purely *subjective*.

<sup>†</sup> ['zweckmässig': the word mystifyingly rendered 'purposive' in translations of Kant's *Critique of Judgement*.

In the quotation which follows, Helmholtz appears to make practical success not a mere criterion of the truth of something, but instead the *meaning* of its being true. However, the questions which introduced the discussion in the present paper ("What is true in our intuition and thought? In what sense do our representations correspond to actuality?") indicate that here he takes *correspondence* to be the meaning of truth, even if practical success is the only criterion for the existence of this correspondence.]

<sup>††</sup> ['unser Erkennen']

<sup>17</sup> This sentence need not be interpreted to mean that Helmholtz maintained the existence of so-called 'innervation sensations', which is mostly denied by modern psychology. He refers only to the clearly indubitable fact, that in executing a voluntary movement we indeed have a consciousness of initiating that movement. We moreover become conscious of alterations in situation and of movements of our limbs themselves by way of the so-called kinaesthetic sensations, which are presumed to be transmitted by special sensory nerves ending in the muscles, sinews and joints.

<sup>18</sup> It is obvious that here Helmholtz even expressly contests – by implication – the existence of special innervation sensations, since his opinion of course is that consciousness of the impulses is not characterised by certain specific qualities of sensation, but by our representations of the movements connected with them. Anyway, the perhaps detailed psychological assumptions made here by Helmholtz are, as also B. Erdmann considers (op. cit., p. 26), "of no significance for the essential content of his theory".

<sup>19</sup> The sense of this and the preceding sentence is probably understandable, but the formulation is again impaired by the tendency to blur the distinction between the sensation as a psychic datum and as an excitation of the sense organ. The contrast made by Helmholtz between the "perceptions of *psychic* activities" and the "sensations of the outer senses" seems only to be possible if the latter are taken to be non-psychic. But this is contradicted by the next two sentences, in which he obviously talks of qualities of sensation as being contents of consciousness. Had Helmholtz contrasted the 'outer' senses with 'inner' (instead of 'psychic') activities, this would have corresponded to the distinction laid down by him above between outer and inner intuition. But this distinction too is dubious on various grounds (compare Schlick, *Allgemeine Erkenntnislehre* [op. cit.], §19). The only legitimate way of doing justice to the distinction developed here by Helmholtz, between spatial and non-spatial experiences, probably consists in classing as the former *all sense perceptions* whatsoever, and as the latter simply all remaining contents of consciousness.

The statement that sensations "proceed subject to some kind of innervation or another" can be elucidated by examples: tactual perceptions generally require a movement of the touching hand, visual perceptions a movement of the seeing eye, of the head, etc. With perceptions of sounds, where this holds to a lesser extent (although even here bending one's ear or approaching towards the source of sound, and so on, play a part), the spatial specificity is also straight away less pronounced.

<sup>20</sup> Here again as above, the expression 'form of intuition' is used in a wholly different sense from that in Kant (compare note 16). But Helmholtz is quite right in coordinating the spatial intuition described by him with the qualities of sensation, since in both cases it is a matter of subjective, psychic contents. What he namely describes, in what precedes and what follows, is *psychological space* (or properly the psychological *spaces* – since one must separate the spatial data of e.g. sensations of movement from those of visual or of tactual perceptions as something wholly different, even though they are all connected by close associations) and not physico-geometrical space. The latter is a non-qualitative, formal conceptual construction; the former, as something intuitively given, is in Helmholtz' words imbued with the qualities of the sensations, and as purely subjective as these are.

<sup>21</sup> In fact, although e.g. the sweet taste of sugar is purely subjective, it is anything but a semblance – as a quality of sensation it is on the contrary something of the most indubi-

table actuality. Likewise the spatial properties of perceptions, with whose help we indeed orientate ourselves in our surroundings with the greatest assurance. A semblance<sup>†</sup> would be present if our perceptions seduced us into mistaken behaviour towards things, as say when we grasp for the mirror image of an object because we falsely take it for the object itself. Compare note 31.

<sup>22</sup> According to this wholly correct explanation, the judgement that outer intuition is spatial is *necessarily* valid, because it is analytic (compare note 1.2). For we term “outside” precisely that which has some spatial specification in the described sense. Naturally, our own body belongs here too, as it is indeed conceived of as something external by contrast with self-consciousness.

<sup>23</sup> The manner of being *a priori* which Helmholtz ascribes, in the words of the text, to space must be termed a ‘psychological’ one. It is not unimportant to ascertain whether, or to what extent, his conception coincides with Kant’s doctrine, to which Helmholtz expressly makes reference and from which he starts.

Now there are two different contrasting interpretations of Kant’s apriorism. Firstly, the one chiefly taken into account by Helmholtz himself (and also advocated e.g. by Schopenhauer), namely the psychological conception of it, which considers the most essential feature of *a priori* cognitions to be their being conditioned by the psychic makeup of the cognising consciousness. Secondly, the transcendental-logical exegesis, according to which the essence of the *a priori* consists in its comprising the ultimate axioms which alone form the foundation for all rigorous cognition and guarantee the latter’s validity. An unbiased reading of Kant’s writings seems to teach us that the second point of view is the more important for his system, while yet appearing as something closely and not quite dissolubly interwoven with the first point of view. Thus neither does the one-sidedly psychological conception do justice to Kant’s thought, nor either the logical interpretation of the so-called ‘neo-Kantian’ schools, which want rigorously to exclude everything psychological from any connexion with the *a priori*.

In Helmholtz’ account, as already said, spatial intuition is acknowledged to be *a priori* only in the psychological sense, so that his epistemology thoroughly deviates from Kant’s, as he himself also plainly perceived. B. Erdmann too, in his last work (*op. cit.* in note 12, p. 27), judges that space is according to Helmholtz a subjective form of intuition “in a sense thoroughly foreign to Kant’s doctrine of space”, and he justly claims that here “Kant’s rationalist thoughts” are “twisted round into their empiricist counterpart”.

<sup>24</sup> A considerable part of the address on ‘The Facts in Perception’ was repeated by Helmholtz word for word in §26 of the second edition of his *Physiologische Optik* [*op. cit.*]. There (p. 588) the present passage is followed by some statements taken over from the first edition, for which room may be found here on account of their epistemological interest:

“As regards, in the first instance, the *properties* of the objects of the external world, a little reflexion shows that all properties ascribed to them by us only characterise *effects* which they exert either upon our senses or upon other objects in nature. Colour, sound, taste, smell, temperature, smoothness and solidity belong to the first class, they characterise effects

<sup>†</sup> [‘Schein’: this word is rendered ‘illusion’ by Kemp Smith in his translation of the *Critique of Pure Reason*, and similarly understood here by M.S. But Helmholtz does not seem to understand by it something necessarily illusory.]

upon our sense organs. Smoothness and solidity characterise the degree of resistance which bodies we contact offer either to a sliding contact or to the pressure of the hand. However, other natural bodies too can take the place of the hand, likewise for examining other mechanical properties, elasticity and weight. Chemical properties as well are related to reactions, i.e. effects which the natural bodies under consideration exert upon others. It is likewise with the other physical properties of bodies – optical, electrical, magnetic.

“Everywhere we deal with mutual relations between one body and another, with effects upon one another which depend on the forces which different bodies exert upon one another. For all natural forces are forces which one body exerts upon the others. If we think of mere matter without forces, it is also without properties – apart from its varied distribution in space and its motion. For that reason too, no property of natural bodies makes its appearance until we involve them in the corresponding mutual effect with other natural bodies or with our own sense organs. But since such a mutual effect can occur at any moment, or as may be can also be brought about at any moment by our will, and we then always see the appropriate kind of mutual effect occur, we ascribe to the objects a lasting capacity for such effects, and one which is always ready to become effective. This lasting capacity we call a *property*’.

“Now it results from this that the *properties* of natural objects, despite this name, in truth characterise nothing whatsoever proper to the individual object in and for itself, but instead always a relation to a second object (which includes our sense organs). The kind of effect must naturally always depend on de peculiarities both of the body exerting an effect and of the body upon which an effect is exerted.

“We do not have even a moment’s doubt about this when speaking about properties of bodies such as emerge if we have two bodies, belonging both to the external world, and one exerts an effect upon the other, e.g. in chemical reactions. Concerning the properties, on the other hand, which are based upon mutual relations between things and our sense organs, people have always tended to forget that here too we have to do with a reaction against a particular reagent, namely our nervous apparatus, and that colour, smell and taste, the feelings of warmth and of cold are also effects which depend quite essentially upon the kind of organ on which an effect is exerted. Of course, the reactions of our senses to natural objects are the most frequently and universally perceived; they have the most predominant importance for our well-being and comfort; the reagent against

<sup>†</sup> [Helmholtz in effect distinguishes three kinds of attribute that bodies may have. There are firstly intrinsic attributes of matter, namely its distribution and motion. These are opposed to properties, as just defined, and the latter are distinguished into those involving a mutual effect between bodies one of which is a sense organ, and those where a sense organ is not one of the bodies concerned.

This recalls Locke’s distinction between primary, secondary and tertiary qualities (*Essay concerning Human Understanding*, book II, ch. 8). The first are “such as are utterly inseparable” from bodies (§9), namely the “bulk, figure, number, situation, and motion or rest of their solid parts” (§23). Secondary qualities of bodies are “powers to produce various sensations in us... by the bulk, figure, texture, and motion of their insensible parts, as colours, sounds, tastes, etc.” (§10). The third kind are powers in a body to change “the bulk, figure, texture, and motion of *another* body, as to make it operate on our senses differently from what it did before” (§23). Locke also complains (as Helmholtz will below) that the similarity between the second and third kinds is commonly overlooked.

A clear difference between their views is that Helmholtz makes fixity (solidity) entirely an attribute of the second kind.]



which we have to try them out is endowed upon us by nature – but this does not change the situation.

“It is for this reason senseless to ask the question of whether cinnabar actually is red as we see it, or whether this is only an illusion of the senses. The sensation of red is the normal reaction of normally formed eyes for light reflected by cinnabar. Someone colour-blind to red will see the cinnabar as black or dark greyish yellow, and this too is the correct reaction for his particular kind of eye. He must only know that his eye is indeed of a different kind from those of other people. The one sensation is not in itself any more correct or false than the other, even if those seeing red have a great majority on their side. In general, the red colour of cinnabar only exists inasmuch as there are eyes constituted in a way similar to those of the majority of people. With equal justice, it is a property of cinnabar to be black, namely for people colour-blind to red. In general, light reflected by cinnabar is not in itself to be called red at all, it is only red for certain kinds of eyes.

“When speaking of properties of bodies which these have in respect of other bodies in the external world, we do not forget to characterise in speech that body too in respect of which the property occurs. We say: ‘Lead is soluble in nitric acid, it is not soluble in sulphuric acid.’ Should we merely want to say ‘Lead is soluble’, we would immediately notice that this is an incomplete assertion, and would immediately have to ask in what it is soluble. But when we say ‘Cinnabar is red’, it is automatically understood implicitly that for our eyes it is red, and for the eyes of other people, which we presuppose to be constituted alike. We believe we need not mention this, and for that reason may well also forget it, and may be misled into believing that redness is a property belonging to cinnabar, or to light reflected from it, quite independent of our sense organs.

“It is something else if we assert that the wavelengths of light reflected from cinnabar have a certain length. That is a statement which we can make independent of the particular nature of our eye. With this statement, however, it is then also only a matter of relations between the substance and the various aether-wave systems.”

<sup>25</sup> In a quite similar manner, H. Poincaré (*Der Wert der Wissenschaft* [‘The value of science’], 2nd ed., pp. 61 f.) elucidates what is peculiar to spatial alterations (movements), as opposed to a qualitative change (e.g. alteration of a body’s colour) in our surroundings.

<sup>26</sup> See e.g. Fichte, *Grundlage der gesamten Wissenschaftslehre* [‘The foundation of the whole theory of science’], §2, no. 9.

<sup>27</sup> Helmholtz speaks, as is customary, simply of ‘spatial intuition’, although – as already remarked (note 20) – one must properly distinguish as many spatial intuitions as there are senses: e.g. the blind man precisely *lacks* intuitive visual space. But there occurs a very close associative connexion between the spatial intuitions, and it may be asked whether one specific sensory domain plays an outstanding role in this, so that it as it were supplies the nucleus of association about which the remaining spatial representations group themselves. One should no doubt answer affirmatively, and it is in the first place – as is also clear from Helmholtz’ own account on pp. 123 f. – the data of sensations of movement to which that central position must be assigned. We may understand Poincaré too in this sense, when he says (*Der Wert der Wissenschaft* [op. cit.], p. 71: “Actual space is the space of movement.” In the second place, as Helmholtz emphasises, one should of course above all take the sense of touch into account, whose representations are moreover most closely connected with those of the sense of movement.

<sup>28</sup> A continuum of  $n$  dimensions is characterised by the fact that it can be completely

split by a continuum of  $(n-1)$  dimensions. Poincaré too (*Wissenschaft und Hypothese* [‘Science and hypothesis’], pp. 33 f.; *Der Wert der Wissenschaft* [op. cit.], p. 73) uses this feature as a characteristic of the  $n$ -dimensional continuum.

<sup>29</sup> This relative clause shows how much the terms taken over from Kant’s philosophy have changed their sense in Helmholtz. According to Kant, all empirical objects are naturally without exception “tied to the form of the human faculty of representation”, since they of course only attain objectivity through precisely this form. Helmholtz here obviously wants to indicate that the spatial properties of bodies indeed have a certain objective significance over and above the “faculty of representation”. This would perhaps correspond to Kant’s opinion insofar as he too held the view that there must for every spatial specification “be also in the object, which is in itself unknown, a ground” (*Metaphysische Anfangsgründe der Naturwissenschaft* [‘Metaphysical principles of natural science’], *Dynamik*, theorem 4, note 2; compare also A. Riehl, *Der philosophische Kriticismus* [‘Philosophical criticism’], 2nd ed., vol I, p. 470).

<sup>30</sup> The formulation of the text, as literally understood, is highly open to attack. It would necessarily lead to misunderstandings and confusions, on grounds already raised more than once (compare note 12), were it not mitigated by the following sentence (see the next note). The *qualities* of sensation certainly belong as such to our consciousness alone, and in no way to the nervous system. If I sense a bitter taste or hear a loud note, my nerves are not then bitter or loud.

Helmholtz expresses himself as if our consciousness with its sensations were located in the nervous system, and consequently also in space – since the nervous system is of course an object in space. One would thus arrive at a ‘projection theory’ of perception, according to which the qualities of sensation are firstly sensed in the body (nervous system) itself, so as thereupon to be “projected out” into space. (This impossible philosophical projection theory should not be confused with a certain purely physiological theory of spatial vision, which – if indeed unsuitably – has likewise been given the name projection theory. Compare on this e.g. von Kries in vol. III of the third edition of Helmholtz’ *Physiologische Optik* [op. cit.], p. 466.) As the spatial (according to Helmholtz himself too) is a form of intuition of our consciousness, it is not proper then to localise our consciousness in turn somewhere in intuitive space.

If one nevertheless attempts this, one gets involved in insoluble contradictions and commits the error which R. Avenarius characterised as ‘introjection’, and which he has illuminated with great acuity – and shown to be avoidable – in his writings *Kritik der reinen Erfahrung* [‘Critique of pure experience’] and *Der menschliche Weltbegriff* [‘The human concept of the world’]. Compare on this Schlick, ‘Idealität des Raumes, Introjektion und psychophysisches Problem’ [‘The ideality of space, introjection and the psychophysical problem’], *Vierteljahresschr. für wiss. Philosophie* 40.

<sup>31</sup> This sentence perhaps does not wholly exclude all misunderstanding. But it still emphasises with welcome plainness that what Helmholtz has just characterised as a semblance – namely that the objects extant in space are imbued with the qualities of sensation – that this is in fact ‘the original truth’, because precisely these qualities actually offer themselves in intuitive space and have an existence nowhere else.

If one understands by ‘bodies’ the objects perceived in intuitive space, we must bear in mind with Kant that they “are not something outside us, but merely representations in us, and hence that it is not the motion of matter that effects representations in us, but that this

motion is instead itself a mere representation" (*Kritik der reinen Vernunft* [op. cit.], 1st ed., p. 387). And in the same sense E. Mach (*Analyse der Empfindungen* ['The analysis of sensations'], 5th ed., p. 23) says: "It is not bodies which generate sensations, but it is sensation complexes instead which form bodies." Thus the 'external causes' of which Helmholtz, on p. 121, considered sensations to be effects, are at any rate not these bodies, but can instead only be understood to be transcendent<sup>†</sup> things (although the concept of a cause cannot, according to Kant, be applied also to the latter). That these things are not subjects of qualities of the senses is something which Helmholtz obviously wanted to stress in the preceding sentence.

It remains somewhat unsatisfying that Helmholtz does not sharply emphasise that there is in *no* way a 'semblance' when we perceive sensory qualities as localised in intuitive space.

<sup>32</sup> We may ask ourselves, on looking back through the last few pages of Helmholtz' account, what constitutes these "most essential features" of spatial intuition. In which case we are probably led to the statement that space is a three-dimensional continuous manifold, in which there is an enduring existence of different things at the same time one beside another, and in which magnitudes can be likened with one another.

<sup>33</sup> Helmholtz here introduces a distinction of the greatest importance for his theory, namely that between the 'general form of spatial intuition' and its 'narrower specifications', which latter are expressed in the axioms of geometry. For him, the general form is that 'schema devoid of any content' which he declared on pp. 1–2 to be the true form of intuition, in respect of which Kant's doctrine of the *a priori* is to be upheld.

One must ask: what then are the *broader* specifications, opposed to these 'narrower' ones, by which the 'general form' is supposed to be characterised? For some or other characteristics must surely belong to it too, since one could not otherwise at all speak of it as of something specific. To this question there seem to be two possible answers. The characteristics sought for might firstly consist of certain peculiarities of sense perceptions which could not be further described but only displayed and witnessed, ones which precisely endow these with the character of spatiality – say the 'extendedness' of a visual perception, or the wholly different 'extendedness' of a tactual perception. While secondly, one might be supposed to look for the demanded specifications of the 'general' form of spatial intuition in precisely those 'most essential features' of which Helmholtz has already spoken, and which were assembled in the preceding note.

In the second case<sup>††</sup>, however, there arises the question of whether those general features could and should, just as well as the 'narrower specifications', be formulated in certain geometrical axioms. Modern geometry is inclined to answer this question affirmatively, and one would then be unable to uphold, in the required sense, the distinction between general and particular specifications of spatial intuition. Thus to make this possible, the

<sup>†</sup> [This term is not a synonym of 'transcendental' (see note 6), but used by Kant for "a principle professing to go beyond the limits of possible experience", see *Kritik der reinen Vernunft* [op. cit.], 2nd ed., p. 352. By a 'transcendent thing' M.S. means an object whose properties might thus purportedly be characterised.]

<sup>††</sup> [which seems to be what Helmholtz intends, since the whole of the previous discussion is supposed to establish the limit up to which "the approach of natural science can take the same path" as Kant.]

'general form' will have to be understood as that indescribable psychological component of spatiality which imbues sense perceptions.

Poincaré (*Der Wert der Wissenschaft* [op. cit.], p. 48) raises the question of whether perhaps space is "a form forced upon our consciousness" as far as its purely *qualitative* specifications are concerned. The properties of this 'general form' or this 'schema devoid of any content' would then have to be expressed in the propositions of *analysis situs* (see above, note I.21). But Poincaré arrives at the result that these propositions too may be presumed to rest upon experiences.

Some neo-Kantians (as P. Natorp, E. Cassirer) have tried to conceive of the *a priori* nature of spatial intuition in the genuine sense of Kant (thus not in Helmholtz' psychological interpretation), but such that it does not comprise the stipulation of some or other specific Euclidean or non-Euclidean geometry. They seem, however, to be defeated by their effective failure to say what are the *a priori* laws of spatial intuition which, in their opinion, then still remain. Compare Schlick, 'Kritizistische oder empiristische Deutung der neuen Physik?' ['A criticist or an empiricist interpretation of the new physics?'], *Kantstudien* 26.

<sup>34</sup> The following basic propositions, however, do not yet form a complete system of axioms upon which the whole of geometry could be built without the assistance of further propositions. See note I.6. Compare with the subsequent account in the text the lecture on 'The Axioms of Geometry'.

<sup>35</sup> Here and in what follows one might more correctly, as before, put '*a priori*' in place of 'transcendental'.

<sup>36</sup> Since Helmholtz speaks here of the form of intuition of the *eye*, it follows that in his opinion too each individual sense has basically its own particular form of intuition, in the significance in which he uses this phrase. It is of interest here, considering what was said in note 33, that von Kries (in the third edition of Helmholtz' *Physiologische Optik* [op. cit.], vol. III, p. 499) has expounded the view that the spatial intuition of the *eye* rests in all of its *quantitative* specifications upon experience, while on the other hand its *qualitative* situational properties (thus the data to be dealt with by *analysis situs*) are given in a fixed manner in advance by physiological laws of formation: "Thus what we can think of as made fixed by laws of formation would still leave alterable the arrangement in the visual field, in a manner similar to that in which a picture painted on a rubber disc can have its shape changed by locally varied stretching of the rubber."

<sup>37</sup> This paragraph explains once more the distinction between what space is as a form of intuition and what the axioms assert about space. Once again Helmholtz' account seems to us to demand the interpretation given in note 33. When we remove from spatial intuition everything that can be expressed conceptually, i.e. in the last analysis by geometrical axioms, there precisely remains only that qualitative element of spatiality (extendedness) which we witness as an ultimate datum not to be analysed further. Helmholtz did not state this himself, and it looks as if he did think of the concept of a form of intuition as endowed with a richer content. But since he nowhere explicitly stated what this content is, it is the task of interpretation to determine the latter such as appears compatible with the psychological and geometrical facts<sup>†</sup>.

<sup>†</sup> [This perhaps goes too far in attempting to 'rescue' Helmholtz. In fact rather than removing everything conceptual from intuition, Helmholtz locates his advance over Kant in having resolved the concept of intuition into elementary thought processes. See the subsequent course of the discussion.]

The most complete statement of what Helmholtz thought of as the content of pure spatial intuition is found – though admittedly only in the form of a comparison – in the following passage (*Wissenschaftliche Abhandlungen*, vol. II, pp. 641 f.), which is taken from the essay directed against Land (see below note 72):

“To recall a quite similar situation, it undoubtedly lies in the makeup of our visual apparatus that everything seen can only be seen as a spatial distribution of colours. That is our innate form of visual perceptions. But this form in no way prejudices how the colours we see are to be spatially ordered one beside another or to follow one another temporally. In the same sense, in my opinion, our representing all external objects in spatial relationships could be the only possible and *a priori* given form in which we can represent objects at all, without this needing to impose any constraint whereby after or beside certain specific spatial perceptions some other specific one must occur, so that e.g. every rectilinear equilateral triangle will have angles of 60 degrees however great its sides may be.

“In Kant, of course, the proof that space is an *a priori* given form of intuition relies essentially upon the belief that the axioms are synthetic propositions given *a priori*. Yet even if one eliminates this proposition and the proof based upon it, the form of spatial intuition could nevertheless still be given *a priori* as the necessary form of intuition of the existence one beside another of different things. In this no essential feature of Kant’s system would be lost. On the contrary, this system would gain in consistency and understandability, because there would then also be eliminated the proof, constructed essentially upon the persuasive power of the axioms of geometry, for the possibility of a metaphysics – of which science Kant himself of course did not know how to discover anything more than the axioms of geometry and natural science. As regards the latter, they are partly of disputed correctness and partly simple inferences from the principle of causality, in other words from the urge of our faculty of understanding to consider everything that occurs to be lawlike, i.e. comprehensible. But since Kant’s critique is otherwise everywhere directed against the admissibility of metaphysical inferences, it seems to me that his system has been freed from an inconsistency, and that a clearer concept of the nature of intuition has been attained, when one gives up the *a priori* origin of the axioms and regards geometry as the first and most perfect of the natural sciences.”

<sup>38</sup> This description does not wholly do justice to Kant’s theory of intuition, since it pays no attention to ‘pure’ intuition, which according to Kant is displayable in empirical intuition as the latter’s form and lawlikeness, and itself is not a ‘psychic process’. Even for Kant it was further resolvable, inasmuch as it indeed splits up into the individual geometrical axioms. Thus when Helmholtz charges Kant with not having tried to resolve intuition further, he must mean something else. He obviously wants to say that Kant failed to ask the question *why* spatial intuition contains in itself precisely the axioms which in fact hold, and not other ones. One must also interpret in this sense the passage in Appendix II (p. 150) where Helmholtz says: “The question of whether intuition was more or less resolvable into conceptual constructions had at that time not yet been raised.”

<sup>39</sup> Compare the lecture on the origin and significance of the axioms of geometry, p. 6.

<sup>40</sup> This sentence has no parallel in the lecture on the axioms, and it raises a significant issue. We show elsewhere (note I.38) that in the case of non-Euclidean geometry the exclusion of ‘every other interpretation’ can strictly speaking never come about by logical compulsion, but instead only by cognitive economy, since besides the geometrical interpretation the physical one always remains possible from a purely theoretical viewpoint.

A full discussion of this matter is found in Helmholtz’ repudiation of Land’s criticism, which is reprinted in this volume (Appendix III).

<sup>41</sup> Concerning the expression “metamathematical” Helmholtz says elsewhere (*Wissenschaftliche Abhandlungen*, vol. II, p. 640): “The name was of course bestowed in an ironical sense by opponents, and modelled upon metaphysics. But since the developers of non-Euclidean geometry have never maintained its objective truth, we may very well accept the name.”

<sup>42</sup> Helmholtz is wholly justified in construing the concept of intuitability, for the present purpose, precisely as is done here, since it is distinct only in degree and not in principle from what he in the next paragraph calls the ‘older concept of intuition’. In these philosophical considerations, one is naturally dealing purely and simply with determining *in principle* the concept of intuition. A thing must count as accessible to intuition, if one can formulate methods with whose help it could be made representable to us in a sensory manner. A person born without eyes cannot in any way learn what a sighted person senses in perceiving yellow: for him there is thus no such method. Or if it were reported that some or other living things possessed a sense organ unknown to us – say for magnetic disturbances – we could in no way whatsoever procure ourselves an experience of the corresponding sensations. These are examples of cases lying beyond all intuitability. The next part of Helmholtz’ account explains that the intuitability of non-Euclidean spaces does not belong to cases of that kind, but instead only requires depicting a succession of perceptions which are put together from purely everyday sensations.

<sup>43</sup> The situation may be illustrated with an example often used by Helmholtz elsewhere. One can form the following inference:

Major premiss: Light which I see with my right eye in the vicinity of my nose, originates from a light source lying to my left.

Minor premiss: When a certain pressure occurs on my right eyeball, I see a patch of light in the direction of my nose.

Conclusion: The source of the light sensation lies to my left (i.e. the pressure lies to my left).

The inference (as is well known, one calls an inference of this form a ‘syllogism’) is false, since it is well known that the eye must be pressed on the right hand side in order that the arising semblance of light should appear to be localised to the left. The mistake arises through one’s falsely considering the major premiss ‘formed from a series of experiences’ to be universally valid, and applying it to a case where it does not hold.

In his *Physiologische Optik* [op. cit.], 2nd ed., p. 582, Helmholtz says: “In millionfold repeated experiences, throughout our whole life, we have found that when we felt an excitation in the nervous apparatuses whose peripheral ends lie on the right hand sides of our two retinas, there lay a luminous object before us to our left. We had to raise our hand to the left in order to mask the light or to grasp the luminous object, or we had to move to the left in order to approach it. Thus although a genuine conscious inference is not present in these cases, the essential and primary task of one is accomplished and its result achieved, if admittedly only through the unconscious processes of the association of representations. This association goes on in the obscure background of our memory, and

its results therefore also force themselves upon our consciousness as if obtained by way of a compelling, seemingly external power, over which our will has no authority. Of course these inductive inferences, which lead to the formation of our sense perceptions, lack the work of purification and examination carried out by conscious thought. As regards their proper nature, nevertheless, I believe I may still term them *inferences* – unconsciously performed inductive inferences.”

<sup>44</sup> On the celebrated theory of ‘unconscious inferences’, which is surveyed in a few words in the paragraph just finished, we shall just briefly make the following comments. Modern psychology energetically rejects the concept of unconscious inference, because it rightly considers thought – the logical process – to be exclusively a function of *consciousness*. It may be asked whether Helmholtz merely uses an unsuitable terminology, or whether the improper terminology is also the expression of thoughts which do not stand up to rigorous epistemological criticism. We believe that Helmholtz’ account, within broad limits, allows the first and favourable interpretation and therefore in fairness calls for it.

In the present paragraph Helmholtz abandons the term ‘unconscious inferences’, admittedly only in order to avoid confusions with Schopenhauer’s ‘wholly unclear’ thoughts. Otherwise he had no substantial reservation about retaining the expression, as emerges from the fact that the passage given in our previous note was taken over unaltered by him from the first edition of his *Physiologische Optik* [op. cit.] in the second edition (1894), and that he expressly added precisely there (p. 602) that he “even now finds the name, up to a certain limit, to be still admissible and significant.”

However, he plainly states that in actuality the process consists of “processes of association”, and also pronounces similarly in other passages, e.g. p. 601 of the 2nd ed. of his *Physiologische Optik*, where he says: “So we see that although this process in its essential parts is brought about – as far as we can discern – only by an involuntary and unconscious action of our memory, it is capable of producing in us representational combinations whose outcome concurs in all essential features with that of conscious thought.” Helmholtz justifies carrying over the logical term to these psychic processes by noting that the latter lead to like *results*, thus render the same services as proper inferences would. Associations and instincts undoubtedly even guide us, in general, more surely in our behaviour towards our environment than does our faculty of understanding. Of course, an inference from like achievement to like nature would not yet on that account be admissible. According to all this, one may well say concerning the questions broached here – where it is a matter of the order of our perceptions – that one need contest only the formulation of Helmholtz’ theory of ‘unconscious inferences’, and not what properly lies at the heart of it.

We shall not investigate here whether this also holds when Helmholtz wants to make unconscious inference responsible not merely for the order of perceptions, but also for the assumption of the existence of bodies as causes of sensations. He does this e.g. in the lecture ‘Über das Sehen des Menschen’ [‘On human vision’] which was held to be the best of the Kant memorial in 1855 in Königsberg. Here he says (*Vorträge und Reden*, 5th ed., vol. I, p. 112): “But if consciousness does not perceive bodies immediately at the places of these bodies themselves, it can only come to an acquaintance with them by an inference.” (Similarly Schopenhauer in *Die Welt als Wille und Vorstellung* [‘The world as will and representation’], §4: “The first, simple, constantly available expression of the understanding is the intuition of the actual world; this is altogether a knowledge of the cause from the effect....”) But since Helmholtz later expressed himself more cautiously on this point (see towards the end of the address on the facts in perception), and since we are concerned not with the historical development of Helmholtz’ views, but with their material and continuing significance, the earlier formulation can be left out of consideration.

More detail on Helmholtz’ theory of unconscious inferences is to be found in B. Erdmann’s repeatedly cited academy essay and in F. Conrat, *H. v. Helmholtz’ psychologische Anschauungen* [‘H. v. Helmholtz’ psychological conceptions’], Halle, 1904, ch. 8.

<sup>45</sup> Lotze regards as local signs of this kind the eye movements (sensations of movement) which are necessary in order to bring the relevant place on the retina to the position of clearest vision. According to W. Wundt (*Physiologische Psychologie* [‘Physiological psychology’], 5th ed., vol. II, pp. 668 ff.), the role of local signs could be played e.g. by differences of colour as well. It is indeed well known that the peripheral parts of the retina e.g. have a quite different colour sensitivity from that of the central ones.

<sup>46</sup> This objection was raised e.g. by du Bois-Reymond after reading the lectures which Helmholtz had given under the title ‘Die neueren Fortschritte in der Theorie des Sehens’ [‘The recent advances in the theory of vision’], and which were reprinted first in the *Preussische Jahrbücher* and then in *Vorträge und Reden*, 5th ed., vol. I, pp. 265ff. In 1868 (see Koenigsberger, *H. v. Helmholtz*, vol. II, p. 84) he wrote to Helmholtz: “It seems to me still to speak against the strictly empiricist conception, that it precisely should be implementable consistently throughout, which – as you concede yourself – is not the case. For if it is innate in the baby calf to go towards the udder on account of the smell, what else might not then be innate in it? To me there seems to remain still so much ineliminable nativism that a bit more or less makes no difference....”

<sup>47</sup> The preceding paragraphs contain a description of the empiricist theory of visual perceptions, as advocated and thus named by Helmholtz. In this description he contrasts its fundamental thoughts clearly and brilliantly with the nativist theory, so that on occasion it might be said (by Fr. Hillebrand in his essay on Ewald Hering, Berlin, 1918, p. 102) of this famous passage that it could “count as the most perfect portrayal of the empiricist theory.”

This is not the place to go into the controversy between the nativist and empiricist views, which even at the present has not exhausted itself. For this is a matter of questions belonging purely to a particular science, and which make no difference as regards the epistemological problem situation. It is true that Helmholtz says (*Physiologische Optik* [op. cit.], 1st ed., p. 796; 2nd ed., p. 945). “In their choice from the various theoretical views, it seems to me... that the various enquirers have been influenced more by a tendency to certain metaphysical ways of regarding things than by the pressure of facts”, and one actually can at times ascertain an influence of the philosophical upon the physiological viewpoint. But it is neither necessary nor justified. The empiricist conception of spatial perception is quite independent of epistemological empiricism, as one may already infer from the fact that e.g. E. Mach combines a far-reaching nativism with a rigorously empiricist – indeed sensualist – epistemology. In fact one cannot see why the nativist assumption, that sensations possess in advance certain spatial properties, should not be just as much compatible with epistemological empiricism as is the fact – which even the latter of course concedes – that sensations are imbued with a quality and modality which is basic and cannot itself be derived.

It must also be stressed emphatically that it is an error to regard nativism as a form of Kantian apriorism, as still sometimes occurs. The purport of Kant’s theory is to explain the apodictic validity of the axioms of geometry (compare note I.3). But nativism is a theory of *sense perception*, its aim cannot then be to give a foundation for a *rigorous* mathematical lawlikeness of space, since all perception as such supplies only approximate data. Thus



for example v. Kries quite correctly says (in the 3rd ed. of Helmholtz' *Physiologische Optik*, vol. III, p. 524) that Hering's nativism "no longer has anything at all in common with Kant's conceptions".

Besides, even within psycho-physiology the opposition of nativist and empiricist conceptions is probably not as fundamental and irreconcilable as is sometimes assumed. A nativist theory which on the issue of localisation does not acknowledge a considerable role to experience as well, seems to be as incapable of implementation as an empiricist one wanting to undertake a construction of spatial representation from purely non-spatial elements of sensation. We also believe that Helmholtz' theory is not to be conceived as an attempt of this latter kind. On the contrary, he indeed considered the spatial to be a form of intuition of a basic kind (in his psychological sense), and we were obliged to interpret this (notes 33 and 37) to mean that every elementary sensation has a component of extendedness (with visual sensations perhaps 'surfacelikeness') which did not arise only at a subsequent stage through experience. Helmholtz' sentence "Everything our eye sees, it sees as an aggregate of coloured surfaces in the visual field – that is its form of intuition" (p. 129) seems in this respect to be wholly conclusive (compare also the quotation given in note 37). As to whether it might prove necessary at some point or another to come still nearer to nativist conceptions, this – as already said – makes no difference at all to the purely epistemological problem of perception.

<sup>48</sup> The task of founding a belief in the existence of the external world upon experiences of the will has been undertaken, in a somewhat different manner, especially by W. Dilthey. See 'Beiträge zur Lösung der Frage vom Ursprung unseres Glaubens an die Realität der Aussenwelt und seinem Recht' [ 'Contributions to solving the question as to the origin of our belief in the reality of the external world and as to its justness' ], *Sitzungsberichte der Berliner Akademie*, 1890.

<sup>49</sup> The most extreme subjective idealism, which denies the existence of an external world different from the subject, is called (as is well known) solipsism. The theoretical possibility of this viewpoint has been conceded by the majority of philosophers, although it has naturally found no serious advocates. This passage indicates that the realist manner of expression in which Helmholtz has clothed his theory of perception does not emanate from an uncritical manner of thought, but is merely chosen as the formulation which most strongly suggests itself, while the realist or idealist interpretation of this formulation can be left to the metaphysician.

<sup>50</sup> Compare e.g. Fichte, *Bestimmung des Menschen* [ 'The determination of man' ], book III, p. I.

<sup>51</sup> In this case the existence of the external world would have to be termed a *fiction* – a thought which H. Vaihinger has developed further in several passages in his *Die Philosophie des Als Ob* [ 'The philosophy of as if' ]. The present passage in Helmholtz seems to have escaped him, since he does not make reference to it in his book. For Helmholtz himself the realist view of the world is naturally not a fiction, but precisely a *hypothesis*.

<sup>52</sup> In this (probably not very apt) extended sense, the term 'thinking' covers all processes of the mind which in some way or another lead to appropriate behaviour towards the external world, and above all its associative and reproductive activities.

<sup>53</sup> The thought expressed here by Helmholtz is a fundamental insight of all epistemology. It lies at the base already of Plato's theory of Ideas – since Plato's Ideas are the unvarying paradigms of the eternally alternating individual things – and likewise at that of the modern theory of science. Stanley Jevons begins his *The Principles of Science* with the sentence: "Science arises from the discovery of Identity amidst Diversity."

<sup>54</sup> The sentence should not be understood literally, since a *law* naturally cannot at all be the object of a perception. Establishing something lawlike is rather always only the termination of a process of observing, ordering and interpreting. Helmholtz only wants to say that detecting laws is something more immediate than ascertaining substances, and it is this thought which is developed in more detail in the next three sentences. If one pursues this thought further, one easily arrives at an insight which is revealing itself more and more plainly in modern natural science, namely that the concept of substance can indeed be wholly reduced to the concept of a law, thus that on the highest level of cognition of nature the former is dispensable as a basic concept.

<sup>55</sup> The surprising manner in which Helmholtz here wishes to define the concept of a cause, by identifying cause and law<sup>†</sup>, would surely be truly inappropriate. For one obviously lacks the right to retain the word 'cause', if one does not use it to characterise a concept concurring at least in its chief features with what one otherwise usually understands by it. If the causes are to be what eternally remains the same, while the effects – the happenings in the world – unremittingly alternate, then one will wholly destroy the reciprocal correspondence between the two, which otherwise belongs to their concepts; an effect could no longer itself be conceived of as the cause of further effects. One cannot see why we should introduce for the concept of a *law* the word cause in addition, which otherwise has a different sense, nor why it should be inappropriate to use the words cause and effect solely for processes in nature, to which their meaning is otherwise restricted in careful scientific usage. It is correct that the term cause is often used in 'a very wishy-washy manner'; but that can be avoided without there having to arise a confusion with the concepts of the antecedent or of the occasion of something. Compare Schlick, *Naturphilosophische Betrachtungen über das Kausalprinzip* [ "Reflections on the principle of causality from the point of view of natural philosophy" ], *Die Naturwissenschaften*, VIII, pp. 461 ff.

<sup>56</sup> This pronouncement might easily be misunderstood without more detailed interpretation. It is probably based in the first place on the thought that if – by a kind of anthropomorphism – we take the concept of a cause (which Helmholtz has indeed just identified with that of a law) to be that of something "compelling the course of natural processes, to be a power equivalent to our will", then it does indeed turn into the concept of force.

In order to characterise Helmholtz' viewpoint further, we may adduce some statements from §5 and §6 of his *Einleitung zu den Vorlesungen über theoretische Physik* [ 'Introduction to the lectures on theoretical physics' ]. There he explains how say the law of attraction initially only reads "Two heavy bodies at a finite distance from each other in space undergo an acceleration, and indeed each of them in the direction of the other", but how we then "by forming abstractions and substituting nouns for the verbs... express it in the form that between any two heavy bodies... there exists continuously a force of attraction of a certain magnitude. We have thereby introduced, in place of the simple

<sup>†</sup> [To be exact, Helmholtz does not intend 'cause' and 'law' to be synonyms, but reserves the former for a very securely established law.]

description of the phenomenon of motion, an abstraction – the force of attraction. We thereby indeed signify nothing – at least nothing still having a factual sense – beyond what is also contained in the description of the mere phenomenon. In laying down the law in this form, which uses the concept of force, we merely add an assurance that this phenomenon of mutual approach of the two bodies occurs – as soon as the conditions for it are given – at any moment of time.”

<sup>57</sup> It is certainly surprising that Helmholtz quotes Fichte relatively frequently and always assentingly, although the rationalist system of this thinker indeed hardly displays any points of contact with his views, and although the form in which Fichte expounded his theoretical ideas certainly could not be to his taste. One can find an explanation only in the fact that Helmholtz, in appraising Fichte, let himself be led by feelings of piety. His sympathy for the philosopher undoubtedly goes back to impressions which he received, according to his own account (*Vorträge und Reden*, 5th ed., vol. I, p. 17), already as a boy in the house of his father, who was an enthusiastic admirer of Fichte.

<sup>58</sup> These famous words of Kirchhoff, which have as it were become a programme for epistemologically oriented physics, are elucidated by him himself (in the foreword to his lectures on mechanics) as follows: “I wish by that to say that it should be a matter only of stating *what* are the appearances that occur, and not of ascertaining their *causes*.” He saw himself forced into this position because the previously customary definition of mechanics was not satisfactory: “One usually defines mechanics as the science of forces, and forces as *causes* which bring about motions or *strive* to bring them about,” and this definition “is infected with the unclarity which cannot be eliminated from the concepts of a cause and of striving.” Kirchhoff’s motive was thus to get away from anthropomorphism, which we spoke of above (note 56).

<sup>59</sup> Let us briefly look at what Helmholtz teaches, in this and the preceding paragraph, about the content and validity of the principle of causality. *Contentually* it expresses, in his (undoubtedly correct) opinion, a trust in the complete comprehensibility of the world; and since this is identical with a trust in all-pervading lawlikeness, “the principle of causality is in fact nothing but the presupposition of the lawlikeness of all of the appearances of nature” (as Helmholtz put it in an addition written in 1881 to his paper on the conservation of force). As regards its *validity* it is a *regulative* principle, which thus serves as a guide line for enquiry, but for which we have no further guarantee than its success. Thus its validity is a factual one in which we must trust, but which we cannot prove.

It is quite different with Kant. For him the law of causality is a *constitutive*<sup>†</sup> principle,

<sup>†</sup> [This is an astonishing statement, since Kant expressly states that the principle of causality is a regulative principle, see *Kritik der reinen Vernunft* [op. cit.], 2nd ed., pp. 221–3. (This is in fact the source of the distinction between constitutive and regulative principles.)

However, Kant means something different from what Helmholtz does by a regulative principle. For Kant this means, in the case of the principle of causality, that we know *a priori* that any given event necessarily has a cause, but do not necessarily know *a priori* what that cause is – this in general has to be ascertained empirically. While an example of a constitutive principle for Kant is that every sensation we experience must have some degree of intensity and that the possible degrees form a continuum; this is a constitutive principle, because it enables one to determine *a priori* what a stronger intensity of a given sensation will be by combining a sufficient number of weaker intensities of that sensation.]

it has a share in the construction of our experience and first makes it possible; it must therefore be necessarily valid for all experiences. According to Kant we do not merely trust in its validity, but are assured of it.

<sup>60</sup> Once again Helmholtz uses the words *a priori* and transcendental in a quite different sense from what Kant does. According to the philosopher, a proposition is called *a priori* if it is *valid*, and can be seen to be valid, independent of experience. And precisely this, according to what Helmholtz declares both before and after this point, is not so with the principle of causality. In using the words *a priori* Helmholtz wants merely to state that the principle of causality cannot be gathered from experience by induction, but instead must always already be presupposed in the interpretation of experiences. But such a presupposition, whose validity is not established in advance, has the character of a *hypothesis*.

Helmholtz also became clear about this, for in a note which he left behind (Koenigsberger, *H. v. Helmholtz*, vol. I, pp. 247f.) he says: “The law of causality (the presupposed lawlikeness of nature) is only a hypothesis, and cannot be proved to be anything other than this. No lawlikeness to date can prove a future lawlikeness. As contrasted with other hypotheses, which express particular laws of nature, the law of causality has an exceptional status only as follows: 1. It is the presupposition for the validity of all the others. 2. It gives us the only possibility whatsoever of knowing something which is unobserved. 3. It is the necessary basis for conduct having a purpose. 4. We are driven to it by the natural mechanics of our representational combinations. Thus we are driven to *wish* it to be correct by the strongest motives; it is the basis of all thought and conduct. Until we have it, we cannot even test it; thus we can only *believe* in it, *conduct* ourselves according to it....” Helmholtz also makes reference to conduct, to the *practical* proof of the principle of causality, in the immediately following statements in the text.

Thus it is not Kant’s viewpoint which he advocates on the issue of the law of causality, but instead he takes the path of David Hume. We may be allowed to add that to us too Hume’s viewpoint seems to be the only one which can withstand all critical attacks.

<sup>61</sup> Contrary to this assurance, it has emerged in our critical comments – and has also indeed already been stressed quite often – that only a few traces of philosophical agreement can be ascertained between Kant and Helmholtz. The doctrine of the subjectivity of spatial intuition and the qualities of the senses, which admittedly was the most important thing for Helmholtz, is properly the only point on which he could justly and unrestrictedly make an appeal to Kant. The explanation why Helmholtz himself believed in a greater agreement than in fact existed is to be found partly in his not always grasping Kant’s doctrine correctly, but instead interpreting it too much in a psychological sense, and partly in that his high esteem for that thinker made the things in common appear to him more important, and the deviations less essential.

<sup>62</sup> In the text, Helmholtz has advocated the opinion that a spatial interpretation is given only to those data of consciousness which alter when body movements are executed. Here he raises against himself the objection that many sensations, originating in the interior of the body, are not noticeably influenced by movements, yet that we nonetheless do not take them to be non-spatial psychic states (like memories or wishes), but rather localise them more or less distinctly at specific places in our bodies. In what follows, the arguments which he brings against this objection are thoroughly suited to removing its force, or at least blunting its edge, and Helmholtz here also has modern psychology essentially on his side. Admittedly, the nativist view that internal sensations are in a certain manner localised

## ON THE ORIGIN AND SIGNIFICANCE OF THE AXIOMS OF GEOMETRY

(*Vorträge und Reden*, 5th ed., vol. II, pp. 1–31)

The fact that a science like geometry can exist, and can be built up in the way it is, has necessarily demanded the closest attention of anyone who ever felt an interest in the fundamental questions of epistemology. There is no other branch of human knowledge which resembles it in having seemingly sprung forth ready-made, like a fully armed Minerva from the head of Jupiter, none before whose devastating aegis dispute and doubt so little dared to lift their eyes. In this it wholly escapes the troublesome and tedious task of gathering empirical facts, as the natural sciences in the narrower sense are obliged to, but instead the form of its scientific procedure is exclusively deduction<sup>1</sup>. Conclusion is developed from conclusion, and yet nobody in his right mind ultimately doubts that these geometrical theorems must have their very practical application to the actuality surrounding us. In surveying and architecture, in mechanical engineering and mathematical physics, we all constantly calculate the most varied kinds of spatial relationships in accordance with geometrical theorems. We expect the issue of their constructions and experiments to be subject to these calculations, and no case is yet known in which we were deceived in this expectation, provided we calculated correctly and with sufficient data.

Thus in the conflict over that issue which forms, as it were, the focus of all the oppositions between philosophical systems, the fact that geometry exists and achieves such things has always been used to prove, as an impressive example, that knowledge of propositions of real content is possible without recourse to a corresponding basis taken from experience. In answering especially Kant's famous question "How are synthetic *a priori* propositions possible?", the axioms of geometry probably constitute the examples which seem to show most evidently, that synthetic propositions *a priori* are in general possible<sup>2</sup>. The circumstance that such propositions exist, and necessarily force our assent, is moreover for him a proof that space is a form, given *a priori*, of all outer intuition<sup>3</sup>. By that he seems to mean not merely that this form given

implications. But if it does, then every congruence proof is supported by a fact drawn only from experience<sup>12</sup>.

I bring up these considerations here in order, in the first place, only to make clear what difficulties we stumble upon, when we analyse fully all of the presuppositions made by us in using the intuitive method. We escape them, if in our investigation of basic principles we employ the analytic method<sup>13</sup> developed in modern calculative geometry. The calculation is wholly carried out as a purely logical operation<sup>14</sup>. It can yield no relationship between the quantities subjected to the calculation, which is not already contained in the equations forming the starting point of the calculation. For this reason, the mentioned recent investigations have been pursued almost exclusively by means of the purely abstract method of analytic geometry.

Yet it is possible besides this to give to some extent an intuitive conception of the points at issue, now that they have been made known by the abstract method. This is best done if we descend into a narrower domain than that of our own spatial world. Let us think of intelligent beings, having only two dimensions, who live and move in the surface of one of our solid bodies – in this there is no logical impossibility. We assume that although they are not capable of perceiving anything outside this surface, they are able to have perceptions, similar to our own, within the expanse of the surface in which they move. When such beings develop their geometry, they will naturally ascribe only two dimensions to their space. They will ascertain that a moving point describes a line and a moving line a plane, this being for them the most complete spatial structure of their acquaintance. But they will as little be able to have any imagination of a further spatial structure that would arise if a surface moved out of their surface-like space, as we are of a structure that would arise if a body moved out of the space known to us.

By the much misused expression 'to imagine' † or 'to be able to think of how something happens', I understand<sup>15</sup> that one could depict the series of sense impressions which one would have if such a thing happened in a particular case. I do not see how one could understand anything else by it without abandoning the whole sense of the expression. But suppose no sense impression whatsoever is known that would

*The Flatland design*

*Flat beings will ascribe two dimensions to space*

*Similarly, we cannot imagine the fourth dimension*

† [See Translator's Note.]

relate to such a never observed process as for us a motion into a fourth dimension of space, or for the surface beings a motion into the third dimension known to us. Then no such 'imagining' is possible, just as little as someone absolutely blind from youth will be able to 'imagine' colours, even if he could be given a conceptual description of them.

The surface beings would besides also be able to draw shortest lines in their surface space. These would not necessarily be straight lines in our sense, but what in geometrical terminology we would call *geodetic* lines of the surface on which they live, ones which will be described by a taut thread applied to the surface and able to slide freely upon it. In what follows, I shall permit myself to term such lines the *straightest* lines<sup>16</sup> of the relevant surface (or of a given space), in order to emphasise the analogy between them and the straight line in a plane. I hope this intuitive expression will make the concept more accessible to my non-mathematical listeners, but without causing confusions.

If moreover beings of this kind lived in an infinite plane, they would lay down precisely our planimetric geometry. They would maintain that only *one* straight line is possible between two points, that through a third point lying outside it only one line parallel to the first can be drawn, that furthermore straight lines can be extended infinitely without their ends meeting again, and so on. Their space might be infinitely extended. But even if they encountered limits to their motion and perception, they would be able to imagine intuitively a continuation beyond those limits. In imagining this, their space would seem to them to be infinitely extended just as ours does to us, although we too cannot leave our earth with our bodies, and our sight only reaches as far as fixed stars are available.

But intelligent beings of this kind could also live in the surface of a sphere. For them, the shortest or straightest line between two points would then be an arc of the great circle through the points in question. Every great circle through two given points is divided thereby into two parts. When their two lengths are not equal, the shorter part is certainly the unique shortest line on the sphere between these two points. But the other and greater arc of the same great circle is also a geodetic or straightest line, meaning that each of its smaller parts is a shortest line between its two endpoints. Because of this circumstance, we cannot simply identify the concept of a geodetic or straightest line with that of a shortest

line. If moreover the two given points are endpoints of the same diameter of the sphere, then any plane through this diameter intersects the surface of the sphere in semicircles, all of which are shortest lines between the two endpoints. So in such a case there are infinitely many shortest lines, all equal to each other, between the two given points. Accordingly, the axiom that only one shortest line exists between two points would not be valid, for the sphere dwellers, without a certain exception.

Parallel lines would be quite unknown to the inhabitants of the sphere. They would maintain that two arbitrary straightest lines, suitably extended, must eventually intersect not in just one point, but in two. The sum of the angles in a triangle would always be greater than two right angles, and would increase with the area of the triangle. For just that reason, they would also lack the concept of geometrical similarity of form between greater and smaller figures of the same kind, since a greater triangle would necessarily have different angles from a smaller one<sup>17</sup>. Their space would be found to be unbounded, yet finitely extended, or at least would have to be imagined to be such.

It is clear that the beings on the sphere, though having the same logical capabilities, would have to lay down a quite different system of geometrical axioms from what the beings in the plane would, and from what we ourselves do in our space of three dimensions. These examples already show us that beings, whose intellectual powers could correspond entirely to our own, would have to lay down different geometrical axioms according to the kind of space in which they lived.

But let us go further, and think of intelligent beings existing in the surface of an egg-shaped body. Between any three points of such a surface one could draw shortest lines, and so construct a triangle. But if one tried to construct congruent triangles at different locations in this surface, it would be found that the angles of two triangles having equally long sides would not turn out to be equal. A triangle drawn at the pointed end of the egg would have angles whose sum differed more from two right angles, than would a triangle with the same sides drawn at the blunt end. It emerges from this, that even such a simple spatial structure as a triangle, in such a surface, could not be moved from one location to another without distortion. It would be found equally, that if circles of equal radii (the length of the radii being always measured by shortest lines along the surface) were constructed at different locations

*No parallel lines*

*Different axioms  
endorsed on the  
sphere*

*The "straightest  
lines" in the  
Flatland will not be  
our "straight lines"*

*The Flatland on  
a Euclidean plane*

*The Flatland on  
a sphere*

*Multiple  
shortest lines*